

# Multris:

## Functional Verification of Multiparty Message Passing in Separation Logic

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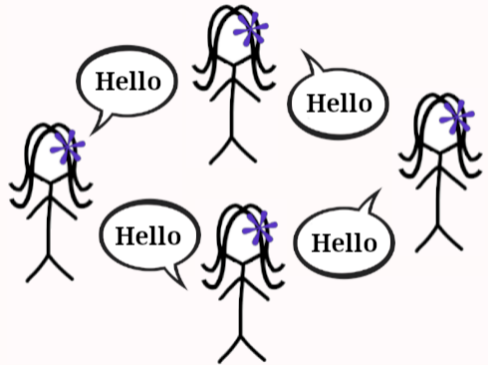
Jules Jacobs

Cornell University

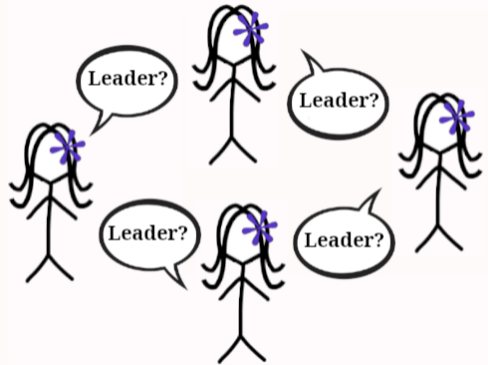
Robbert Krebbers

Radboud University  
Nijmegen

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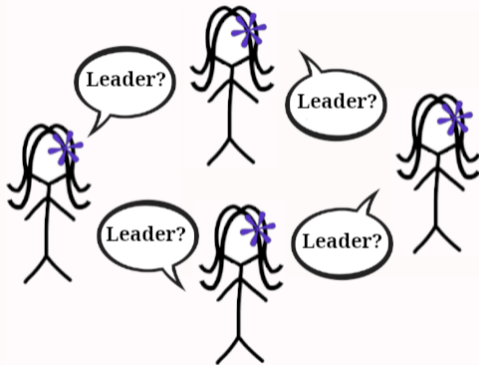
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- ▶ Message passing with dependent interactions between multiple parties



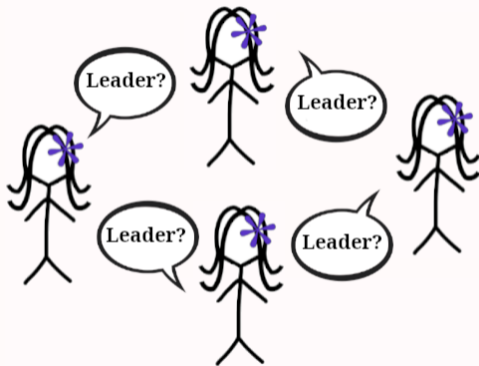
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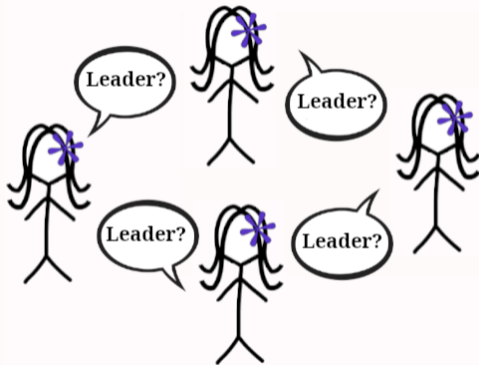
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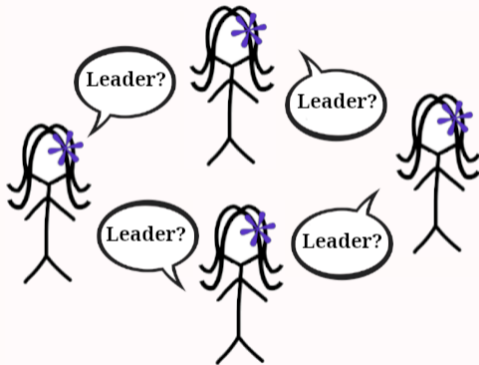
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## Warrants functional verification

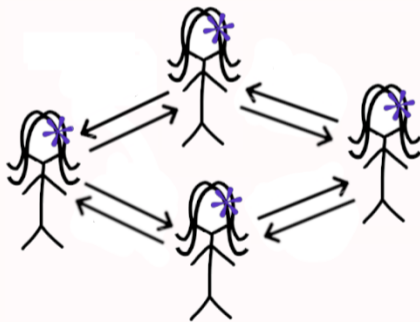
- ▶ No results that supports all the above
- ▶ We want validation in a mechanised theorem prover



# Our Setting of Multiparty Message Passing

**Message passing over bi-directional channels with distinct channel endpoints**

- ▶ Each endpoint correspond to one party





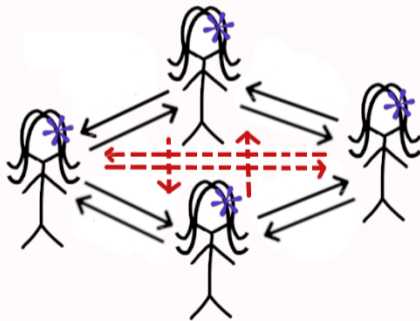
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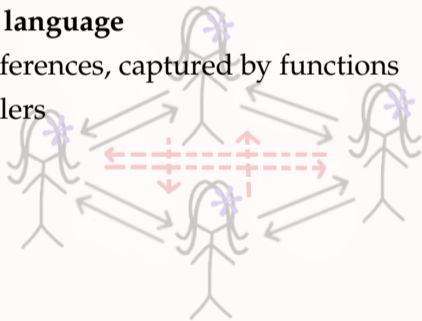
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**Channel endpoints are first class citizens of the language**

- ▶ Can be passed around as values, stored in references, captured by functions
- ▶ Similar to Go channels and BSD socket handlers



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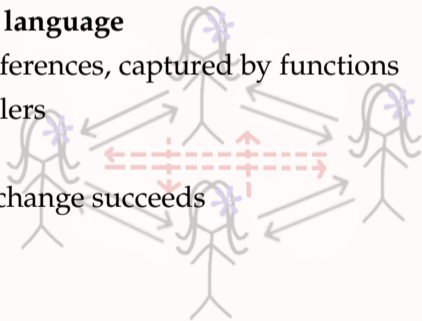
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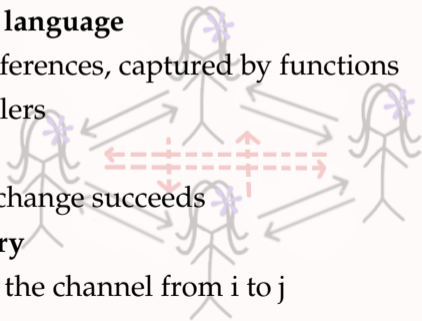
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**Implementation via references in shared memory**

- ▶ Implemented as an  $N \times N$  matrix where  $i,j$  is the channel from  $i$  to  $j$



# Multiparty Message Passing in Shared Memory

## Multiparty channels API:

- new\_chan( $n$ )**      Creates a multiparty channel with  $n$  parties, returning a tuple  $(c_0, \dots, c_{(n-1)})$  of endpoints
- $c_i[j].\mathbf{send}(v)$       Sends a value  $v$  via endpoint  $c_i$  to party  $j$  (synchronously)
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## Example program: Roundtrip

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let  $(c_0, c_1, c_2) = \mathbf{new\_chan}(3)$  in  
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# Safety and Functional Correctness

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**Goal:** Prove crash-freedom (safety) and verify asserts (functional correctness)

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<b>Binary</b>	Session Types	Dependent separation protocols

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**Key Idea:** Define and prove consistency via separation logic!

## **Multiparty dependent separation protocols (MDSPs)**

- ▶ Rich specification language for describing multiparty message passing
- ▶ Protocol consistency defined in terms of semantic duality, proven in separation logic

## **Multris separation logic**

- ▶ Separation logic for verifying multiparty communication via MDSPs
- ▶ Support for language-parametric instantiation of Multris

## **Verification of suite of multiparty programs**

- ▶ Increasingly intricate variations of the roundtrip program
- ▶ Chang and Roberts ring leader election algorithm

## **Full mechanisation in Rocq**

- ▶ With tactic support for protocol consistency and channel primitives

# Roadmap of this talk

## **Separation Logic Primer**

- ▶ Operational semantics
- ▶ Hoare triples
- ▶ Separation logic

## **Tour of the Multris separation logic**

- ▶ Multiparty dependent separation protocols and protocol consistency
- ▶ Verification rules for multiparty channels
- ▶ Verification of suite of roundtrip variations

## **Conclusion and Future Work**

# Separation Logic Primer

**HeapLang:** Untyped OCaml-like language

$$v, w \in \text{Val} ::= z \mid \mathbf{true} \mid \mathbf{false} \mid () \mid \ell \mid \lambda x. e$$
$$e \in \text{Expr} ::= v \mid x \mid e_1 e_2 \mid \mathbf{let} x = e_1 \mathbf{in} e_2 \mid e_1; e_2 \mid$$
$$\mathbf{ref} e \mid !e \mid e_1 \leftarrow e_2 \mid$$
$$(e_1 \parallel e_2) \mid \mathbf{assert}(e) \mid \dots$$

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**Example program:**

```
let  $\ell_1 = \mathbf{ref} 0$  in  
let  $\ell_2 = \mathbf{ref} 0$  in  
  ( $\ell_1 \leftarrow !\ell_1 + 2 \parallel \ell_2 \leftarrow !\ell_2 + 2$ );  
assert( $!\ell_1 + !\ell_2 = 4$ )
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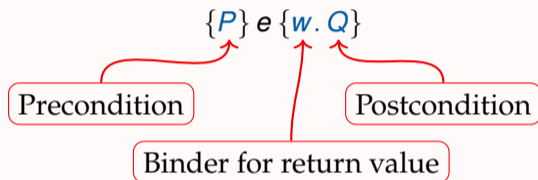
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  ( $\ell_1 \leftarrow !\ell_1 + 2 \parallel \ell_2 \leftarrow !\ell_2 + 2$ );  
assert( $!\ell_1 + !\ell_2 = 4$ )
```

**Goal:** Prove crash-freedom (safety) and verify asserts (functional correctness)

# Hoare Triples

**Hoare triples** for partial functional correctness:



If the initial state satisfies  $P$ , then:

- ▶ **Safety:**  $e$  does not crash
- ▶ **Postcondition validity:** if  $e$  terminates with value  $v$ , then the final state satisfies  $Q[v/w]$

# Separation Logic

**Separation logic:** propositions assert ownership and knowledge about the state

**The points-to connective:**  $\ell \mapsto v$

- ▶ Provides the knowledge that location  $\ell$  has value  $v$ , and
- ▶ Provides **exclusive ownership** of  $\ell$

**Separating conjunction:**  $P * Q$  captures that the state consists of disjoint parts satisfying  $P$  and  $Q$ .

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**Separating conjunction:**  $P * Q$  captures that the state consists of disjoint parts satisfying  $P$  and  $Q$ .

Enables modular reasoning, through disjointness:

$$\frac{\text{HT-FRAME} \quad \{P\} e \{w. Q\}}{\{P * R\} e \{w. Q * R\}}$$

# Hoare Triples for Separation Logic

## Hoare triples for references:

HT-ALLOC

$\{\text{True}\} \mathbf{ref} v \{l. l \mapsto v\}$

HT-LOAD

$\{l \mapsto v\} !l \{w. w = v * l \mapsto v\}$

HT-STORE

$\{l \mapsto v\} l \leftarrow w \{l \mapsto w\}$

# Hoare Triples for Separation Logic

## Hoare triples for references:

$$\begin{array}{ccc} \text{HT-ALLOC} & \text{HT-LOAD} & \text{HT-STORE} \\ \{ \text{True} \} \mathbf{ref} v \{ l. l \mapsto v \} & \{ l \mapsto v \} ! l \{ w. w = v * l \mapsto v \} & \{ l \mapsto v \} l \leftarrow w \{ l \mapsto w \} \end{array}$$

## Hoare triples for structural expressions:

$$\begin{array}{ccc} \text{HT-LET} & & \text{HT-ASSERT} \\ \frac{\{ P \} e_1 \{ w_1. Q \} \quad \forall w_1. \{ Q \} e_2 [w_1/x] \{ w_2. R \}}{\{ P \} \mathbf{let} x = e_1 \mathbf{in} e_2 \{ w_2. R \}} & & \frac{\{ P \} e \{ w. w = \mathbf{true} * Q \}}{\{ P \} \mathbf{assert}(e) \{ Q \}} \end{array}$$

$$\begin{array}{ccc} \text{HT-SEQ} & & \text{HT-PAR} \\ \frac{\{ P \} e_1 \{ w_1. Q \} \quad \forall w_1. \{ Q \} e_2 \{ w_2. R \}}{\{ P \} e_1; e_2 \{ w_2. R \}} & & \frac{\{ P_1 \} e_1 \{ Q_1 \} \quad \{ P_2 \} e_2 \{ Q_2 \}}{\{ P_1 * P_2 \} (e_1 \parallel e_2) \{ Q_1 * Q_2 \}} \end{array}$$

# Example Program - Verified

```
let  $l_1$  = ref 0 in  
let  $l_2$  = ref 0 in  
( $l_1 \leftarrow !l_1 + 2 \parallel l_2 \leftarrow !l_2 + 2$ );  
assert(! $l_1 + !l_2 = 4$ )
```

# Example Program - Verified

```
{True}  
let  $l_1 = \mathbf{ref} 0$  in  
let  $l_2 = \mathbf{ref} 0$  in  
 $(l_1 \leftarrow !l_1 + 2 \parallel l_2 \leftarrow !l_2 + 2);$   
assert $(!l_1 + !l_2 = 4)$   
{True}
```



# Example Program - Verified

```
{True}
let  $l_1 = \text{ref } 0$  in      // HT-LET, HT-ALLOC
{ $l_1 \mapsto 0$ }
let  $l_2 = \text{ref } 0$  in
( $l_1 \leftarrow !l_1 + 2 \parallel l_2 \leftarrow !l_2 + 2$ );
assert( $!l_1 + !l_2 = 4$ )
{True}
```

# Example Program - Verified

```
{True}
let  $l_1 = \mathbf{ref} 0$  in      // HT-LET, HT-ALLOC
{ $l_1 \mapsto 0$ }
let  $l_2 = \mathbf{ref} 0$  in      // HT-LET, HT-ALLOC, HT-FRAME
{ $l_1 \mapsto 0 * l_2 \mapsto 0$ }
( $l_1 \leftarrow !l_1 + 2 \parallel l_2 \leftarrow !l_2 + 2$ );
assert( $!l_1 + !l_2 = 4$ )
{True}
```

# Example Program - Verified

```
{True}
let  $l_1 = \text{ref } 0$  in      // HT-LET, HT-ALLOC
{ $l_1 \mapsto 0$ }
let  $l_2 = \text{ref } 0$  in      // HT-LET, HT-ALLOC, HT-FRAME
{ $l_1 \mapsto 0 * l_2 \mapsto 0$ }
( $\left( \begin{array}{c} \{l_1 \mapsto 0\} \\ l_1 \leftarrow !l_1 + 2 \end{array} \parallel \begin{array}{c} \{l_2 \mapsto 0\} \\ l_2 \leftarrow !l_2 + 2 \end{array} \right)$ ); // HT-SEQ, HT-PAR
assert( $!l_1 + !l_2 = 4$ )
{True}
```

# Example Program - Verified

```
{True}
let  $l_1 = \mathbf{ref} 0$  in      // HT-LET, HT-ALLOC
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# Example Program - Verified

```
{True}
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{ $l_1 \mapsto 0$ }
let  $l_2 = \mathbf{ref} 0$  in      // HT-LET, HT-ALLOC, HT-FRAME
{ $l_1 \mapsto 0 * l_2 \mapsto 0$ }
 $\left( \begin{array}{c|c} \{l_1 \mapsto 0\} & \{l_2 \mapsto 0\} \\ l_1 \leftarrow !l_1 + 2 & l_2 \leftarrow !l_2 + 2 \\ \{l_1 \mapsto 2\} & \{l_2 \mapsto 2\} \end{array} \right);$  // HT-SEQ, HT-PAR, HT-LOAD, HT-STORE
{ $l_1 \mapsto 2 * l_2 \mapsto 2$ }
assert( $!l_1 + !l_2 = 4$ )
{True}
```

# Example Program - Verified

```
{True}
let  $l_1 = \mathbf{ref} 0$  in      // HT-LET, HT-ALLOC
{ $l_1 \mapsto 0$ }
let  $l_2 = \mathbf{ref} 0$  in      // HT-LET, HT-ALLOC, HT-FRAME
{ $l_1 \mapsto 0 * l_2 \mapsto 0$ }
 $\left( \begin{array}{c|c} \{l_1 \mapsto 0\} & \{l_2 \mapsto 0\} \\ l_1 \leftarrow !l_1 + 2 & l_2 \leftarrow !l_2 + 2 \\ \{l_1 \mapsto 2\} & \{l_2 \mapsto 2\} \end{array} \right);$  // HT-SEQ, HT-PAR, HT-LOAD, HT-STORE
{ $l_1 \mapsto 2 * l_2 \mapsto 2$ }
assert( $!l_1 + !l_2 = 4$ )    // HT-LOAD, HT-ASSERT
{True}
```

# But What About Multiparty Channels?

**Roundtrip program:**

```
let (c0, c1, c2) = new_chan(3) in  
( let x = 40 in c0[1].send(x); assert(c0[2].recv() = x + 2) ||| let y = c1[0].recv() in c1[2].send(y + 1) ||| let z = c2[1].recv() in c2[0].send(z + 1) )
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**Goal:** Prove crash-freedom (safety) and verify asserts (functional correctness)

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  c2[0].send(z + 1) )
```

**Goal:** Prove crash-freedom (safety) and verify asserts (functional correctness)

**Sub-Goal:** Hoare triples for multiparty channel primitives

HT-NEW

{???} **new\_chan**(n) {???}

HT-SEND

{???} c[i].**send**(v) {???}

HT-RECV

{???} c[i].**recv**() {???}



# Tour of Multris

**Channel endpoint ownership:  $c \rightsquigarrow p$**

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**Protocols:**  $![i] (\vec{x}:\vec{\tau}) \langle v \rangle . p \mid ?[i] (\vec{x}:\vec{\tau}) \langle v \rangle . p \mid \mathbf{end}$

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**Example:**  $![1] (x : \mathbb{Z}) \langle x \rangle . ?[2] \langle x + 2 \rangle . \mathbf{end}$

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**Example:**  $![1] (x : \mathbb{Z}) \langle x \rangle . ?[2] \langle x + 2 \rangle . \mathbf{end}$

**Rules:**

HT-NEW

$\{\text{CONSISTENT } \vec{p} * |\vec{p}| = n + 1\} \mathbf{new\_chan}(|\vec{p}|) \{(c_0, \dots, c_n) . c_0 \rightsquigarrow \vec{p}_0 * \dots * c_n \rightsquigarrow \vec{p}_n\}$

**Channel endpoint ownership:**  $c \rightsquigarrow p$

**Protocols:**  $![i] (\vec{x} : \vec{\tau}) \langle v \rangle . p \mid ?[i] (\vec{x} : \vec{\tau}) \langle v \rangle . p \mid \mathbf{end}$

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**Rules:**

HT-NEW

$\{\text{CONSISTENT } \vec{p} * |\vec{p}| = n + 1\} \mathbf{new\_chan}(|\vec{p}|) \{(c_0, \dots, c_n). c_0 \rightsquigarrow \vec{p}_0 * \dots * c_n \rightsquigarrow \vec{p}_n\}$

HT-SEND

$\{c \rightsquigarrow ![i] (\vec{x} : \vec{\tau}) \langle v \rangle . p\} c[i].\mathbf{send}(v[\vec{t}/\vec{x}]) \{c \rightsquigarrow p[\vec{t}/\vec{x}]\}$

**Channel endpoint ownership:**  $c \rightsquigarrow p$

**Protocols:**  $![i] (\vec{x} : \vec{\tau}) \langle v \rangle . p \mid ?[i] (\vec{x} : \vec{\tau}) \langle v \rangle . p \mid \mathbf{end}$

**Example:**  $![1] (x : \mathbb{Z}) \langle x \rangle . ?[2] \langle x + 2 \rangle . \mathbf{end}$

**Rules:**

HT-NEW

$\{\text{CONSISTENT } \vec{p} * |\vec{p}| = n + 1\} \mathbf{new\_chan}(|\vec{p}|) \{(c_0, \dots, c_n). c_0 \rightsquigarrow \vec{p}_0 * \dots * c_n \rightsquigarrow \vec{p}_n\}$

HT-SEND

$\{c \rightsquigarrow ![i] (\vec{x} : \vec{\tau}) \langle v \rangle . p\} c[i].\mathbf{send}(v[\vec{t}/\vec{x}]) \{c \rightsquigarrow p[\vec{t}/\vec{x}]\}$

HT-RECV

$\{c \rightsquigarrow ?[i] (\vec{x} : \vec{\tau}) \langle v \rangle . p\} c[i].\mathbf{recv}() \{w. \exists \vec{t}. w = v[\vec{t}/\vec{x}] * c \rightsquigarrow p[\vec{t}/\vec{x}]\}$

# Protocol Consistency

For any synchronised exchange from  $i$  to  $j$ , given the binders of  $i$ , we must:

1. Instantiate the binders of  $j$
2. Prove equality of exchanged values
3. Prove protocol consistency where  $i$  and  $j$  are updated to their respective tails

Repeat until no more synchronised exchanges exist.



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Repeat until no more synchronised exchanges exist.

$$\frac{(\forall i, j. \text{semantic\_dual } \vec{p} i j)}{\text{CONSISTENT } \vec{p}}^*$$
$$\frac{\vec{p}_i = ![j] (x_1 : \tau_1) \langle v_1 \rangle . p_1 \quad * \quad \vec{p}_j = ?[i] (x_2 : \tau_2) \langle v_2 \rangle . p_2 \quad *}{\forall x_1 : \tau_1. \exists x_2 : \tau_2. v_1 = v_2 \quad * \quad \triangleright (\text{CONSISTENT } (\vec{p}[i := p_1][j := p_2]))}^*}{\text{semantic\_dual } \vec{p} i j}$$

# Protocol Consistency - Example

Protocol consistency example:

$$\begin{aligned}\vec{p}_0 &:= ![1] (x : \mathbb{Z}) \langle x \rangle. ?[2] \langle x + 2 \rangle. \mathbf{end} \\ \vec{p}_1 &:= ?[0] (y : \mathbb{Z}) \langle y \rangle. ![2] \langle y + 1 \rangle. \mathbf{end} \\ \vec{p}_2 &:= ?[1] (z : \mathbb{Z}) \langle z \rangle. ![0] \langle z + 1 \rangle. \mathbf{end}\end{aligned}$$

Protocol consistency:

$$\frac{(\forall i, j. \text{semantic\_dual } \vec{p} \ i \ j)}{\text{CONSISTENT } \vec{p}}^*$$
$$\frac{\vec{p}_i = ![j] (x_1 : \vec{\tau}_1) \langle v_1 \rangle. p_1 \ * \ \vec{p}_j = ?[i] (x_2 : \vec{\tau}_2) \langle v_2 \rangle. p_2 \ * \ \forall x_1 : \vec{\tau}_1. \exists x_2 : \vec{\tau}_2. v_1 = v_2 \ * \ \triangleright (\text{CONSISTENT } (\vec{p}[i := p_1][j := p_2]))}{\text{semantic\_dual } \vec{p} \ i \ j}^*$$

# Roundtrip Example - Verified

Roundtrip program:

```
let (c0, c1, c2) = new_chan(3) in  
( let x = 40 in c0[1].send(x); assert(c0[2].recv() = x + 2) ||| let y = c1[0].recv() in c1[2].send(y + 1) ||| let z = c2[1].recv() in c2[0].send(z + 1) )
```

Protocols:

```
c0 ↦ ![1] (x : ℤ) ⟨x⟩. ?[2] ⟨x + 2⟩. end  
c1 ↦ ?[0] (y : ℤ) ⟨y⟩. ![2] ⟨y + 1⟩. end  
c2 ↦ ?[1] (z : ℤ) ⟨z⟩. ![0] ⟨z + 1⟩. end
```

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```
let (c0, c1, c2) = new_chan(3) in  
( let x = 40 in c0[1].send(x);  
  assert(c0[2].recv() = x + 2) ||| c1[2].send(y + 1) ||| let z = c2[1].recv() in )  
  c2[0].send(z + 1)
```

Protocols:

$$\begin{aligned}c_0 &\rightsquigarrow ![1] (x : \mathbb{Z}) \langle x \rangle. ?[2] \langle x + 2 \rangle. \mathbf{end} \\c_1 &\rightsquigarrow ?[0] (y : \mathbb{Z}) \langle y \rangle. ![2] \langle y + 1 \rangle. \mathbf{end} \\c_2 &\rightsquigarrow ?[1] (z : \mathbb{Z}) \langle z \rangle. ![0] \langle z + 1 \rangle. \mathbf{end}\end{aligned}$$

Verified Functional Correctness!

# Roundtrip Reference Example

**Roundtrip reference program:**

```
let (c0, c1, c2) = new_chan(3) in  
  ( let x = 40 in           || let ℓ = c1[0].recv() in || let ℓ = c2[0].recv() in )  
  ( let ℓ = ref x in       || ℓ ← (!ℓ + 1);           || ℓ ← (!ℓ + 1);           )  
  ( c0[1].send(ℓ);         || c1[2].send(ℓ)           || c2[0].send(ℓ)           )  
  ( c0[2].recv();         ||                               ||                               )  
  ( assert(!ℓ = x + 2)    ||                               ||                               )
```

**Goal:** Prove crash-freedom (safety) and verify asserts (functional correctness)

# Multris with Resources

**Protocols:**  $![i] (\vec{x}:\vec{\tau})\langle v \rangle \{P\}.p \mid ?[i] (\vec{x}:\vec{\tau})\langle v \rangle \{P\}.p$

**Example:**  $![1] (\ell : \text{Loc}, x : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto x \}. ?[2] \langle () \rangle \{ \ell \mapsto (x + 2) \}.$  **end**

**Rules:**

HT-NEW

$\{ \text{CONSISTENT } \vec{p} * |\vec{p}| = n + 1 \}$  **new\_chan** $(|\vec{p}|) \{ (c_0, \dots, c_n). c_0 \multimap \vec{p}_0 * \dots * c_n \multimap \vec{p}_n \}$

HT-SEND

$\{ c \multimap ![i] (\vec{x}:\vec{\tau})\langle v \rangle \{P\}.p * P[\vec{t}/\vec{x}] \}$   $c[i].\mathbf{send}(v[\vec{t}/\vec{x}]) \{ c \multimap p[\vec{t}/\vec{x}] \}$

HT-RECV

$\{ c \multimap ?[i] (\vec{x}:\vec{\tau})\langle v \rangle \{P\}.p \}$   $c[i].\mathbf{recv}() \{ w. \exists \vec{t}. w = v[\vec{t}/\vec{x}] * c \multimap p[\vec{t}/\vec{x}] * P[\vec{t}/\vec{x}] \}$

# Protocol Consistency with Resources

For any synchronised exchange from  $i$  to  $j$ , given the binders and resources of  $i$ , we must:

1. Instantiate the binders of  $j$
2. Prove equality of exchanged values and the resources of  $j$
3. Prove protocol consistency where  $i$  and  $j$  are updated to their respective tails

Repeat until no more synchronised exchanges exist.

$$\frac{(\forall i, j. \text{semantic\_dual } \vec{p} i j)}{\text{CONSISTENT } \vec{p}}^*$$
$$\frac{\vec{p}_i = ![j] (\vec{x}_1 : \vec{\tau}_1) \langle v_1 \rangle \{P_1\}. p_1 \text{ } * \vec{p}_j = ? [i] (\vec{x}_2 : \vec{\tau}_2) \langle v_2 \rangle \{P_2\}. p_2 \text{ } * \forall \vec{x}_1 : \vec{\tau}_1. P_1 \text{ } * \exists \vec{x}_2 : \vec{\tau}_2. v_1 = v_2 \text{ } * P_2 \text{ } * \triangleright (\text{CONSISTENT } (\vec{p}[i := p_1][j := p_2]))}{\text{semantic\_dual } \vec{p} i j}^*$$

# Protocol Consistency with Resources - Example

**Protocol consistency example:**

$$\begin{aligned}\vec{p}_0 &:= ![1] (l : \text{Loc}, x : \mathbb{Z}) \langle l \rangle \{l \mapsto x\}. ?[2] \langle () \rangle \{l \mapsto (x + 2)\}. \text{end} \\ \vec{p}_1 &:= ?[0] (l : \text{Loc}, y : \mathbb{Z}) \langle l \rangle \{l \mapsto y\}. ![2] \langle l \rangle \{l \mapsto (y + 1)\}. \text{end} \\ \vec{p}_2 &:= ?[1] (l : \text{Loc}, z : \mathbb{Z}) \langle l \rangle \{l \mapsto z\}. ![0] \langle () \rangle \{l \mapsto (z + 1)\}. \text{end}\end{aligned}$$

**Protocol consistency:**

$$\frac{(\forall i, j. \text{semantic\_dual } \vec{p} \ i \ j)}{\text{CONSISTENT } \vec{p}}^*$$
$$\frac{\forall \vec{x}_1 : \vec{\tau}_1. P_1 * \exists \vec{x}_2 : \vec{\tau}_2. v_1 = v_2 * P_2 * \triangleright (\text{CONSISTENT } (\vec{p}[i := p_1][j := p_2]))}{\text{semantic\_dual } \vec{p} \ i \ j}^*$$



# Roundtrip Reference Example - Verified

Roundtrip reference program:

```
let (c0, c1, c2) = new_chan(3) in  
  ( let x = 40 in           || let ℓ = c1[0].recv() in || let ℓ = c2[0].recv() in )  
  ( let ℓ = ref x in       || ℓ ← (!ℓ + 1);           || ℓ ← (!ℓ + 1);           )  
  ( c0[1].send(ℓ);         || c1[2].send(ℓ)           || c2[0].send(ℓ)           )  
  ( c0[2].recv();         ||                               ||                               )  
  ( assert(!ℓ = x + 2)    ||                               ||                               )
```

Protocols:

$c_0 \rightsquigarrow ! [1] (\ell : \text{Loc}, x : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto x \}. ? [2] \langle () \rangle \{ \ell \mapsto (x + 2) \}. \text{end}$

$c_1 \rightsquigarrow ? [0] (\ell : \text{Loc}, y : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto y \}. ! [2] \langle \ell \rangle \{ \ell \mapsto (y + 1) \}. \text{end}$

$c_2 \rightsquigarrow ? [1] (\ell : \text{Loc}, z : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto z \}. ! [0] \langle () \rangle \{ \ell \mapsto (z + 1) \}. \text{end}$

**Goal:** Prove crash-freedom (safety) and verify asserts (functional correctness)

# Protocol Consistency - Recursion

**Protocols are contractive in the tail:**

$\mu rec. ! [1] (l : \text{Loc}, x : \mathbb{Z}) \langle l \rangle \{l \mapsto x\}. ? [2] \langle () \rangle \{l \mapsto (x + 2)\}. rec$

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Protocol consistency example:

$$\vec{p}_0 = \mu rec. ! [1] (\ell : \text{Loc}, x : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto x \}. ? [2] \langle () \rangle \{ \ell \mapsto (x + 2) \}. rec$$

$$\vec{p}_1 = \mu rec. ? [0] (\ell : \text{Loc}, y : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto y \}. ! [2] \langle \ell \rangle \{ \ell \mapsto (y + 1) \}. rec$$

$$\vec{p}_2 = \mu rec. ? [1] (\ell : \text{Loc}, z : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto z \}. ! [0] \langle () \rangle \{ \ell \mapsto (z + 1) \}. rec$$

Recursion via Löb induction  $\triangleright$

$$\frac{\vec{p}_i = ! [j] (\vec{x}_1 : \vec{\tau}_1) \langle v_1 \rangle \{ P_1 \}. p_1 * \vec{p}_j = ? [i] (\vec{x}_2 : \vec{\tau}_2) \langle v_2 \rangle \{ P_2 \}. p_2 * \forall \vec{x}_1 : \vec{\tau}_1. P_1 * \exists \vec{x}_2 : \vec{\tau}_2. v_1 = v_2 * P_2 * \triangleright (\text{CONSISTENT } (\vec{p}[i := p_1][j := p_2]))}{\text{semantic\_dual } \vec{p} i j} *$$

# Protocol Consistency - Framing

Consider the replacement of process 1 with a forwarder:

```
let  $v = c_1[0].\text{recv}()$  in  $c_1[1].\text{send}(v)$ 
```

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$$\vec{p}_0 = \mu\text{rec}. ![1] (\ell : \text{Loc}, x : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto x \}. ?[2] \langle () \rangle \{ \ell \mapsto (x + 1) \}. \text{rec}$$
$$\vec{p}_1 = \mu\text{rec}. ?[0] (\ell : \text{Loc}, y : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto y \}. ![2] \langle \ell \rangle \{ \ell \mapsto y \}. \text{rec}$$
$$\vec{p}_2 = \mu\text{rec}. ?[1] (\ell : \text{Loc}, z : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto z \}. ![0] \langle () \rangle \{ \ell \mapsto (z + 1) \}. \text{rec}$$

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$$\vec{p}_0 = \mu\text{rec}. ! [1] (\ell : \text{Loc}, x : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto x \}. ? [2] \langle () \rangle \{ \ell \mapsto (x + 1) \}. \text{rec}$$
$$\vec{p}_1 = \mu\text{rec}. ? [0] (v : \text{Val}) \langle v \rangle. ! [2] \langle v \rangle. \text{rec}$$
$$\vec{p}_2 = \mu\text{rec}. ? [1] (\ell : \text{Loc}, z : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto z \}. ! [0] \langle () \rangle \{ \ell \mapsto (z + 1) \}. \text{rec}$$

# Protocol Consistency - Framing

Consider the replacement of process 1 with a forwarder:

$$\text{let } v = c_1[0].\text{recv}() \text{ in } c_1[1].\text{send}(v)$$

Protocol consistency example:

$$\vec{p}_0 = \mu \text{rec}. ![1] (\ell : \text{Loc}, x : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto x \}. ?[2] \langle () \rangle \{ \ell \mapsto (x + 1) \}. \text{rec}$$
$$\vec{p}_1 = \mu \text{rec}. ?[0] (v : \text{Val}) \langle v \rangle. ![2] \langle v \rangle. \text{rec}$$
$$\vec{p}_2 = \mu \text{rec}. ?[1] (\ell : \text{Loc}, z : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto z \}. ![0] \langle () \rangle \{ \ell \mapsto (z + 1) \}. \text{rec}$$

Protocol consistency owns resources while in transit:

$$\frac{\vec{p}_i = ![j] (\vec{x}_1 : \vec{\tau}_1) \langle v_1 \rangle \{ P_1 \}. p_1 * \vec{p}_j = ?[i] (\vec{x}_2 : \vec{\tau}_2) \langle v_2 \rangle \{ P_2 \}. p_2 * \forall \vec{x}_1 : \vec{\tau}_1. P_1 * \exists \vec{x}_2 : \vec{\tau}_2. v_1 = v_2 * P_2 * \triangleright (\text{CONSISTENT} (\vec{p}[i := p_1][j := p_2]))}{\text{semantic\_dual } \vec{p} i j} *$$

# Protocol Consistency - Branching

Consider the extension of process 1 with a rerouter:

```
let  $(v, b) = c_1[0].\text{recv}()$  in  $c_1[\text{if } b \text{ then } 2 \text{ else } 3].\text{send}(v)$ 
```



# Protocol Consistency - Branching

Consider the extension of process 1 with a rerouter:

```
let (v, b) = c1[0].recv() in c1[if b then 2 else 3].send(v)
```

Protocol consistency example:

$$\begin{aligned}\vec{p}_0 &= \mu rec. ![1] (\ell : \text{Loc}, x : \mathbb{Z}, b : \mathbb{B}) \langle (\ell, b) \rangle \{ \ell \mapsto x \}. \\ &\quad ?[\text{if } b \text{ then } 2 \text{ else } 3] \langle () \rangle \{ \ell \mapsto (x + 1) \}. rec \\ \vec{p}_1 &= \mu rec. ?[0] (v : \text{Val}, b : \mathbb{B}) \langle (v, b) \rangle. ![\text{if } b \text{ then } 2 \text{ else } 3] \langle v \rangle. rec \\ \vec{p}_2, \vec{p}_3 &= \mu rec. ?[1] (\ell : \text{Loc}, z : \mathbb{Z}) \langle \ell \rangle \{ \ell \mapsto z \}. ![0] \langle () \rangle \{ \ell \mapsto (z + 1) \}. rec\end{aligned}$$

We can do case analysis on the binders:

$$\frac{\vec{p}_i = ![j] (\vec{x}_1 : \vec{\tau}_1) \langle v_1 \rangle \{ P_1 \}. p_1 * \vec{p}_j = ?[i] (\vec{x}_2 : \vec{\tau}_2) \langle v_2 \rangle \{ P_2 \}. p_2 * \forall \vec{x}_1 : \vec{\tau}_1. P_1 * \exists \vec{x}_2 : \vec{\tau}_2. v_1 = v_2 * P_2 * \triangleright (\text{CONSISTENT } (\vec{p}[i := p_1][j := p_2]))}{\text{semantic\_dual } \vec{p} i j}^*$$

# Language Parametricity of Multris

# Multris Ghost Theory

We defined the MDSP's via Iris's recursive domain equation solver and proved language-generic ghost theory rules based on Iris's ghost state machinery

PROTO-ALLOC

$$\frac{\text{CONSISTENT } \vec{p}}{\models \exists \chi. \text{prot\_ctx } \chi \mid \vec{p} \mid * \bigstar_{i \mapsto p \in \vec{p}} \text{prot\_own } \chi \ i \ p}$$

PROTO-VALID

$$\frac{\text{prot\_ctx } \chi \ n \quad \text{prot\_own } \chi \ i \ p}{i < n}$$

PROTO-STEP

$$\frac{\text{prot\_own } \chi \ i \ (! [j] (\vec{x}_1 : \vec{\tau}_1) \langle v_1 \rangle \{P_1\}. p_1) \quad \text{prot\_ctx } \chi \ n \quad P_1[\vec{t}_1 / \vec{x}_1] \quad \text{prot\_own } \chi \ j \ (? [i] (\vec{x}_2 : \vec{\tau}_2) \langle v_2 \rangle \{P_2\}. p_2)}{\models \triangleright \exists (\vec{t}_2 : \vec{\tau}_2). \text{prot\_ctx } \chi * \text{prot\_own } \chi \ i \ (p_1[\vec{t}_1 / \vec{x}_1]) * \text{prot\_own } \chi \ j \ (p_2[\vec{t}_2 / \vec{x}_2]) * (v_1[\vec{t}_1 / \vec{x}_1]) = (v_2[\vec{t}_2 / \vec{x}_2]) * P_2[\vec{t}_2 / \vec{x}_2]}$$

One can then define language-specific  $c \mapsto p$  and prove Hoare triple rules (such as HT-SEND, HT-RECV, and HT-NEW) for a given language using the ghost theory

# Conclusion and Future Work

## **Dependent multiparty protocols are non-trivial to prove sound**

- ▶ Mismatched dependencies (quantifiers) makes syntatic analysis difficult
- ▶ Fullfillment of received resources is tricky

## **Concurrent separation logic is a good fit for multiparty protocols**

- ▶ Quantifier scopes enable inherent tracking of dependencies
- ▶ Separation logic enables framing of resources

## **Mechanisation yields crucial level of automation**

- ▶ Imperative for non-trivial multiparty protocol consistency proofs

## **Additional features**

- ▶ Asynchronous communication/subprotocols
- ▶ Mixed choice

## **Semantic Multiparty Session Type System**

- ▶ Investigate correspondences with syntactic protocol consistency

## **Better methodology for proving protocol consistency**

- ▶ Abstraction and Modularity via separation logic

## **Deadlock freedom guarantees**

- ▶ Leverage connectivity graphs for multiparty communication

## **Multris for distributed systems**

- ▶ Leverage the Aneris separation logic

## **Additional features**

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## **Multris for distributed systems**

- ▶ Leverage the Aneris separation logic

## **And much more!**

$!$ [1]  $\langle$ “Thank you” $\rangle$ {MultriSOverview}.  
 $\mu$ rec. ?[1] ( $q$  : Question  $i$ )  $\langle$  $q$  $\rangle$ {AboutMultriS  $q$ }.  
    ! $[i]$  ( $a$  : Answer)  $\langle$  $a$  $\rangle$ {Insightful  $q$   $a$ }.rec