Verified Message-Passing Concurrency in Iris

Separation Logic Meets Session Types

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Tutorial Timeline

Part 1: 14:00 – 15:30

- Introduction (10 min)
- Layered implementation of session channels (10 min)
- Basic concurrent separation logic and one-shot protocols (30 min)
- **Break** (10 min)
- Dependent separation protocols (30 min)

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Part 2: 16:00 – 17:30

- Iris invariants and ghost state (30 min)
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- Supervised Coq hacking (50 min)

Message Passing Concurrency

Shared-memory message passing concurrency:

- Structured approach to concurrent programming
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Bi-directional session channels:

new()	Create channel and return two endpoints c1 and c2
c.**send**(*v*)	Send value *v* over endpoint *c*
c.**recv**()	Receive and return next inbound value on endpoint *c*

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Example program:

$$\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in}\;\left(\begin{array}{l} c_1.\mathbf{send}(40); \\ \mathbf{let}\,y = c_1.\mathbf{recv}()\,\mathbf{in} \\ \mathbf{assert}(y = 42) \end{array} \;\middle\|\; \begin{array}{l} \mathbf{let}\,x = c_2.\mathbf{recv}()\,\mathbf{in} \\ c_2.\mathbf{send}(x + 2) \end{array}\right)$$

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Actris: Session type-based extension of Iris

- **Session type-based:** Reasoning about message-passing concurrency via dependent separation protocols
- **MiniActris:** Layered minimalistic version of Actris from first principles (ICFP'23 Functional Pearl)

Learning Goals of this Tutorial

After this tutorial you will be able to:

- Design layers of abstractions in concurrent separation logic
- Verify sample programs using these abstractions
- Verify these abstractions using the Iris methodology
- Mechanize these results using the Iris Proof Mode in Coq

Overview of Abstraction Layers

Layer	**Reasoning principles / specifications**
#1 Iris's HeapLang	Basic concurrent separation logic Iris invariants and ghost state
#2 One-shot channels	One-shot protocols
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Layered implementation of session channels ala. MiniActris

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Layer #1: Iris's Heap Lang

Untyped OCaml-like language with

- Mutable references
- Higher-order recursive functions
- Parallel composition-based concurrency
- Assert statements

$$
\begin{aligned}
v, w \in \mathsf{Val} ::= {} & z \mid \mathbf{true} \mid \mathbf{false} \mid () \mid \ell \mid && (z \in \mathbb{Z}, \ell \in \mathsf{Loc}) \\
& \mathbf{rec}\ f\ x = e \mid (v, w) \mid \mathbf{Some}\, v \mid \mathbf{None} \\
e \in \mathsf{Expr} ::= {} & v \mid x \mid e_1\ e_2 \mid \\
& \mathbf{fst}\, e \mid \mathbf{snd}\, e \mid \\
& (\mathbf{match}\, e\, \mathbf{with}\, \mathbf{Some}\, v \Rightarrow e_1 \mid \mathbf{None} \Rightarrow e_2\, \mathbf{end}) \mid \\
& \mathbf{ref}\, e \mid \mathbf{free}\, e \mid !\, e \mid e_1 \leftarrow e_2 \mid \\
& (e_1 \parallel e_2) \mid \mathbf{assert}(e) \mid \ldots
\end{aligned}
$$

Example Program – Sequential

Simple sequential program:

$$
\begin{array}{l}
\texttt{ref\_prog} \triangleq \\
\quad \textbf{let}\ \ell = \textbf{ref}\ \textbf{None}\ \textbf{in} \\
\quad \ell \leftarrow \textbf{Some}\,42; \\
\quad \textbf{let}\ x = !\,\ell\ \textbf{in} \\
\quad \textbf{free}\ \ell; \\
\quad \textbf{assert}(x = \textbf{Some}\,42)
\end{array}
$$

The **`assert`** statement halts the program if the condition does not reduce to **`true`**

Layer #2: One-Shot Channels

One-shot channel implementation:

$$
\begin{aligned}
\texttt{new1}\,() &\triangleq \texttt{ref None}\\
\texttt{send1}\,c\,v &\triangleq c \leftarrow \texttt{Some}\,v\\
\texttt{recv1}\,c &\triangleq \texttt{let}\,x = !\,c\,\texttt{in}\\
&\quad\ \texttt{match}\,x\,\texttt{with}\\
&\quad\ \ \texttt{None} \Rightarrow \texttt{recv1}\,c\\
&\quad\ \mid \texttt{Some}\,v \Rightarrow \texttt{free}\,c;\ v\\
&\quad\ \texttt{end}
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&\qquad \mid \mathbf{Some}\,v \Rightarrow \mathbf{free}\,c;\ v \\
&\qquad \mathbf{end}
\end{aligned}
$$

Concurrent program that uses one-shot channels:

$$
\begin{aligned}
&\mathtt{oneshot\_prog} \triangleq \\
&\quad \mathbf{let}\,c = \mathbf{new1}\,()\ \mathbf{in} \\
&\quad \left(\begin{array}{l|l} \mathbf{send1}\,c\,42 & \mathbf{let}\,x = \mathbf{recv1}\,c\ \mathbf{in} \\ & \mathbf{assert}(x = 42) \end{array}\right)
\end{aligned}
$$

Example Programs – Reference Passing

Passing references over one-shot channels:

$$
\begin{array}{l}
\texttt{oneshot\_ref\_prog} \triangleq \\
\quad \mathbf{let}\, c = \mathbf{new1}\,() \,\mathbf{in} \\
\quad \left(\begin{array}{l|l}
\mathbf{let}\, \ell = \mathbf{ref}\, 42 \,\mathbf{in} & \mathbf{let}\, \ell = \mathbf{recv1}\, c \,\mathbf{in} \\
\mathbf{send1}\, c\, \ell & \mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
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\end{array} \right)
\end{array}
$$

Example Programs – Higher-Order Channels

Passing one-shot channels over one-shot channels:

$$
\begin{array}{l}
\texttt{oneshot\_chan\_prog} \triangleq \\
\quad \mathbf{let}\, c = \mathbf{new1}\,() \,\mathbf{in} \\
\quad \left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\mathbf{let}\, c' = \mathbf{new1}\,() \,\mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right) \;\Big\|\; \left(\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c \,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\, ()
\end{array}\right)
\end{array}
$$

Layer #3: Functional Session Channels

Implementation (inspired by Kobayashi et al., Dardha et al.):

$$\mathbf{new}_{\mathbf{fun}}\,() \triangleq \mathbf{new1}\,()$$

$$\mathbf{send}\,c\,v \triangleq \mathbf{let}\,c' = \mathbf{new1}\,()\,\mathbf{in}\,\mathbf{send1}\,c\,(v, c');\ c' \qquad \mathbf{close}\,c \triangleq \mathbf{send1}\,c\,()$$

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Recovering the one-shot channel example:

$$\texttt{ses\_fun\_ref\_prog} \triangleq \mathbf{let}\,c = \mathbf{new}_{\mathbf{fun}}\,()\,\mathbf{in} \begin{pmatrix} \mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\ \mathbf{let}\,c' = \mathbf{send}\,c\,\ell\,\mathbf{in} \\ \mathbf{wait}\,c'; \\ \mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\ \mathbf{assert}(x = 42) \end{pmatrix} \Bigg\| \begin{pmatrix} \mathbf{let}\,(\ell, c') = \mathbf{recv}\,c\,\mathbf{in} \\ \ell \leftarrow (!\,\ell + 2); \\ \mathbf{close}\,c' \end{pmatrix}$$

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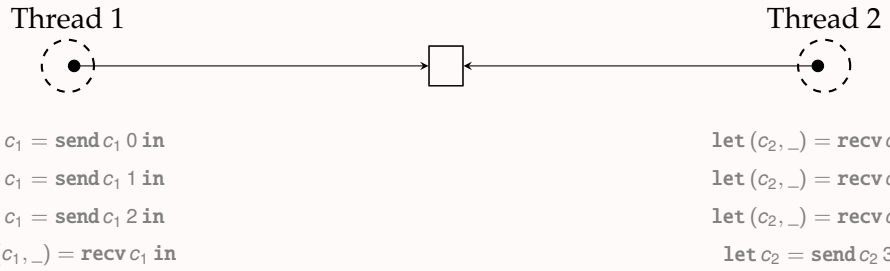
Emerging polarized bi-directional linked list:



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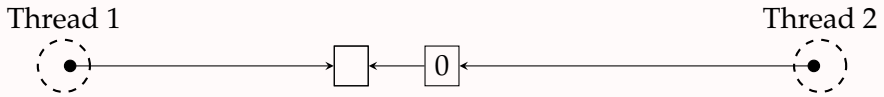

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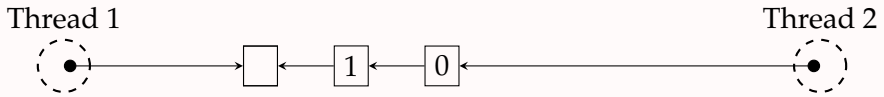

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Layer #3: Functional Session Channels – Intuition

Implementation (inspired by Kobayashi et al., Dardha et al.):

$$\mathtt{new}_{\mathtt{fun}}\,() \triangleq \mathtt{new1}\,()$$

$$\mathtt{send}\,c\,v \triangleq \mathtt{let}\,c' = \mathtt{new1}\,()\,\mathtt{in}\,\mathtt{send1}\,c\,(v, c');\ c' \qquad \mathtt{close}\,c \triangleq \mathtt{send1}\,c\,()$$

$$\mathtt{recv}\,c \triangleq \mathtt{recv1}\,c \qquad \mathtt{wait}\,c \triangleq \mathtt{recv1}\,c$$

Emerging polarized bi-directional linked list:

$\mathtt{let}\,c_1 = \mathtt{send}\,c_1\,0\,\mathtt{in}$

$\mathtt{let}\,c_1 = \mathtt{send}\,c_1\,1\,\mathtt{in}$

$\mathtt{let}\,c_1 = \mathtt{send}\,c_1\,2\,\mathtt{in}$

$\mathtt{let}\,(c_1, \_) = \mathtt{recv}\,c_1\,\mathtt{in}$

$\mathtt{let}\,(c_2, \_) = \mathtt{recv}\,c_2\,\mathtt{in}$

$\mathtt{let}\,(c_2, \_) = \mathtt{recv}\,c_2\,\mathtt{in}$

$\mathtt{let}\,(c_2, \_) = \mathtt{recv}\,c_2\,\mathtt{in}$

$\mathtt{let}\,c_2 = \mathtt{send}\,c_2\,3\,\mathtt{in}$

Layer #4: Session Channels

Session channel implementation:

$$\mathbf{new}\,() \triangleq \mathbf{let}\,c = \mathbf{new}_{\mathbf{fun}}\,()\,\mathbf{in}\,(\mathbf{ref}\,c, \mathbf{ref}\,c)$$

$$c.\mathbf{send}(v) \triangleq c \leftarrow \mathbf{send}\,(!\,c)\,v \qquad c.\mathbf{close}() \triangleq \mathbf{close}\,(!\,c); \mathbf{free}\,c$$

$$c.\mathbf{recv}() \triangleq \mathbf{let}\,(v, c') = \mathbf{recv}\,!\,c\,\mathbf{in}\,c \leftarrow c'; v \qquad c.\mathbf{wait}() \triangleq \mathbf{wait}\,(!\,c); \mathbf{free}\,c$$

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Session channel example:

$$\texttt{ses\_ref\_prog} \triangleq \mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \begin{pmatrix} \mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\ c_1.\mathbf{send}(\ell); c_1.\mathbf{wait}(); \\ \mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\ \mathbf{assert}(x = 42) \end{pmatrix} \Bigg\| \begin{pmatrix} \mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\ \ell \leftarrow (!\,\ell + 2); c_2.\mathbf{close}() \end{pmatrix}$$

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Goal: Verify this example and all its dependencies in Iris

Questions?

Basic concurrent separation logic and one-shot protocols

Tutorial Timeline

Part 1: 14:00 – 15:30

- ~~Introduction (10 min)~~
- ~~Layered implementation of session channels (10 min)~~
- Basic concurrent separation logic and one-shot protocols (30 min)
- **Break** (10 min)
- Dependent separation protocols (30 min)

Break (30 min)

Part 2: 16:00 – 17:30

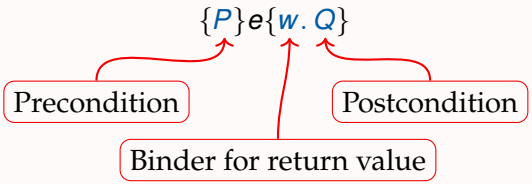
- Iris invariants and ghost state (30 min)
- **Break** (10 min)
- Supervised Coq hacking (50 min)

Overview of Abstraction Layers

Layer	**Reasoning principles / specifications**
#1 Iris's HeapLang	Basic concurrent separation logic Iris invariants and ghost state
#2 One-shot channels	One-shot protocols
#3 Functional session channels	Dependent separation protocols
#4 Session channels	Dependent separation protocols

Hoare Triples

Hoare triples for partial functional correctness:



If the initial state satisfies P, then:

- **Safety:** e does not crash
- **Postcondition validity:** if e terminates with value v, then the final state satisfies $Q[v/w]$

We often write $\{P\}\ e\ \{Q\} \triangleq \{P\}\ e\ \{w.\, w = () * Q\}$

Separation Logic [O'Hearn, Reynolds, Yang 2001]

Separation logic propositions assert ownership of resources

The points-to connective $\ell \mapsto v$

- Provides the knowledge that location ℓ has value v, and
- Provides exclusive ownership of ℓ

Separating conjunction $P * Q$ captures that the state consists of disjoint parts satisfying P and Q.

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Separating conjunction $P * Q$ captures that the state consists of disjoint parts satisfying P and Q.

Enables modular reasoning, through disjointness:

$$\textsc{Ht-frame}\ \frac{\{P\}\ e\ \{w.\ Q\}}{\{P * R\}\ e\ \{w.\ Q * R\}}$$

Sample of Separation Logic Rules

Heap manipulation:

HT-ALLOC
$\{\mathsf{True}\}\ \mathbf{ref}\, v\ \{w.\ \exists \ell.\ w = \ell * \ell \mapsto v\}$

HT-LOAD
$\{\ell \mapsto v\}\ !\,\ell\ \{w.\ w = v * \ell \mapsto v\}$

HT-STORE
$\{\ell \mapsto v\}\ \ell \leftarrow w\ \{\ell \mapsto w\}$

HT-FREE
$\{\ell \mapsto v\}\ \mathbf{free}\, \ell\ \{\mathsf{True}\}$

Structural and general rules:

HT-LET
$$\frac{\{P\}\ e_1\ \{w_1.\ Q\} \qquad \forall w_1.\ \{Q\}\ e_2[w_1/x]\ \{w_2.\ R\}}{\{P\}\ \mathbf{let}\, x = e_1\, \mathbf{in}\, e_2\ \{w_2.\ R\}}$$

HT-VAL
$\{\mathsf{True}\}\ v\ \{w.\ w = v\}$

HT-SEQ
$$\frac{\{P\}\ e_1\ \{w_1.\ Q\} \qquad \forall w_1.\ \{Q\}\ e_2\ \{w_2.R\}}{\{P\}\ e_1; e_2\ \{w_2.\ R\}}$$

HT-ASSERT
$$\frac{\{P\}\ e\ \{w.\ w = \mathbf{true} * Q\}}{\{P\}\ \mathbf{assert}(e)\ \{Q\}}$$

Simple Verification Example – Sequential Reference Program

$$
\begin{array}{l}
\mathbf{let}\ \ell = \mathbf{ref}\ \mathbf{None}\ \mathbf{in} \\
\ell \leftarrow \mathbf{Some}\ 42; \\
\mathbf{let}\ x = !\,\ell\ \mathbf{in} \\
\mathbf{free}\ \ell; \\
\mathbf{assert}\ (x = \mathbf{Some}\ 42)
\end{array}
$$

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$\{\mathsf{True}\}$
let $\ell = \mathbf{ref}\ \mathbf{None}\ \mathbf{in}$
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Simple Verification Example – Sequential Reference Program

$\{\mathsf{True}\}$
let $\ell = \mathbf{ref}\ \mathbf{None}\ \mathbf{in}$ // Ht-let, Ht-alloc
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let $x = !\,\ell\ \mathbf{in}$ // Ht-let, Ht-load
$\{\ell \mapsto 42 * x = \mathbf{Some}\ 42\}$
free ℓ;
assert $(x = \mathbf{Some}\ 42)$

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free $\ell;$ // Ht-seq, Ht-free
$\{x = \mathbf{Some}\ 42\}$
assert $(x = \mathbf{Some}\ 42)$ // Ht-assert
$\{\mathsf{True}\}$

One-Shot Channel Specifications

Channel ownership $c \rightarrowtail p$

- Provides exclusive permission to use the channel c according to the protocol p

Protocols and duality:

$$\textbf{Protocols:} \quad p ::= (\mathsf{Send}, \Phi) \mid (\mathsf{Recv}, \Phi) \quad \text{where} \quad \Phi : \mathsf{Val} \to \mathsf{Prop}$$

$$\textbf{Duality:} \quad \overline{(\mathsf{Send}, \Phi)} \triangleq (\mathsf{Recv}, \Phi) \qquad \overline{(\mathsf{Recv}, \Phi)} \triangleq (\mathsf{Send}, \Phi)$$

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One-shot channel specifications:

HT-NEW
$\{\mathsf{True}\}\ \mathbf{new1}\ ()\ \{w.\ \exists c.\ w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

HT-SEND
$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}\ \mathbf{send1}\ c\ v\ \{\mathsf{True}\}$

HT-RECV
$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}\ \mathbf{recv1}\ c\ \{w.\ \Phi\ w\}$

Concurrency Specifications

Parallel composition rule:

$$\textsc{Ht-par}\ \frac{\{P_1\}\ e_1\ \{w_1.Q_1\} \qquad \{P_2\}\ e_2\ \{w_2.Q_2\}}{\{P_1 * P_2\}\ (e_1 \parallel e_2)\ \{w.\ \exists w_1, w_2.\ w = (w_1, w_2) * Q_1 * Q_2\}}$$

One-Shot Channel Verification Examples – Basics

$$
\begin{array}{l}
\mathbf{let}\, c = \mathbf{new1}\,() \,\mathbf{in} \\
\left(\begin{array}{l|l}
\mathbf{send1}\, c\, 42 & \mathbf{let}\, x = \mathbf{recv1}\, c \,\mathbf{in} \\
 & \mathbf{assert}(x = 42)
\end{array} \right)
\end{array}
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One-shot protocol:

$$\mathtt{prot} \triangleq (\mathsf{Send}, \lambda w.\, w = 42)$$

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One-shot reference protocol:

$$\texttt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

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$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,() \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{ref\_prot}\} & \{c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 42 \,\mathbf{in} & \mathbf{let}\, \ell = \mathbf{recv1}\, c \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * \ell \mapsto 42\} & \{\ell \mapsto 42\} \\
\mathbf{send1}\, c\, \ell & \mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\{\mathsf{True}\} & \{x = 42\} \\
 & \mathbf{assert}(x = 42) \\
 & \{\mathsf{True}\}
\end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,() \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{ref\_prot}\} & \{c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 42 \,\mathbf{in} & \mathbf{let}\, \ell = \mathbf{recv1}\, c \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * \ell \mapsto 42\} & \{\ell \mapsto 42\} \\
\mathbf{send1}\, c\, \ell & \mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\{\mathsf{True}\} & \{x = 42\} \\
 & \mathbf{assert}(x = 42) \\
 & \{\mathsf{True}\}
\end{array}\right) \\
\{\mathsf{True}\}
\end{array}
$$

One-shot reference protocol:

$$\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\mathbf{let}\, c = \mathbf{new1}\,() \,\mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\mathbf{let}\, c' = \mathbf{new1}\,() \,\mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c \,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\,()
\end{array}
\right)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right) \;\Big\|\; \left(\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\, ()
\end{array}\right)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right. \quad \Bigg\| \quad \left(\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\, ()
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\, \ell})$$

$$\mathtt{chan\_prot}'\, (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\, \ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right) \\
\quad \Big\| \\
\left(\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\, \ell + 2); \\
\mathbf{send1}\, c'\, ()
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathsf{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathsf{chan\_prot}'\, \ell})$$

$$\mathsf{chan\_prot}'\, (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathsf{chan\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right) \;\Big\|\; \left(\begin{array}{l}
\{c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\, ()
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathsf{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathsf{chan\_prot}'\, \ell})$$
$$\mathsf{chan\_prot}'\, (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathsf{chan\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right. \\
\left\|\begin{array}{l}
\{c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\, ()
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathsf{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathsf{chan\_prot}'\, \ell})$$
$$\mathsf{chan\_prot}'\, (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathsf{chan\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\{c' \rightarrowtail (\mathsf{chan\_prot}'\ \ell)\} \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right. \\
\left\| \begin{array}{l}
\{c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\, ()
\end{array}\right.
\end{array}
$$

One-shot channel protocols:

$$\mathsf{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathsf{chan\_prot}'\ \ell})$$
$$\mathsf{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathsf{chan\_prot}\} & \{c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} & \mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\, \ell + 2); \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); & \mathbf{send1}\, c'\, () \\
\{c' \rightarrowtail (\mathsf{chan\_prot}'\, \ell)\} & \\
\mathbf{recv1}\, c'; & \\
\{\ell \mapsto 42\} & \\
\mathbf{let}\, x = !\, \ell\, \mathbf{in}\, \mathbf{free}\, \ell; & \\
\mathbf{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathsf{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathsf{chan\_prot}'\, \ell})$$
$$\mathsf{chan\_prot}'\, (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathsf{chan\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\{c' \rightarrowtail (\mathsf{chan\_prot}'\ \ell)\} \\
\mathbf{recv1}\, c'; \\
\{\ell \mapsto 42\} \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42)
\end{array}\right. \quad \Bigg\| \quad \left.\begin{array}{l}
\{c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\, ()
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathsf{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathsf{chan\_prot}'\ \ell})$$
$$\mathsf{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathsf{chan\_prot}\} & \{c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} & \mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); & \mathbf{send1}\, c'\, () \\
\{c' \rightarrowtail (\mathsf{chan\_prot}'\ \ell)\} & \\
\mathbf{recv1}\, c'; & \\
\{\ell \mapsto 42\} & \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell; & \\
\{x = 42\} & \\
\mathbf{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathsf{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathsf{chan\_prot}'\ \ell})$$
$$\mathsf{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\, c = \textbf{new1}\,()\, \textbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{chan\_prot}\} & \{c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\textbf{let}\, \ell = \textbf{ref}\, 40\, \textbf{in} & \textbf{let}\, (\ell, c') = \textbf{recv1}\, c\, \textbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} & \{c' \rightarrowtail \overline{(\mathtt{chan\_prot}'\ \ell)} * \ell \mapsto 40\} \\
\textbf{let}\, c' = \textbf{new1}\,()\, \textbf{in}\, \textbf{send1}\, c\, (\ell, c'); & \ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell)\} & \textbf{send1}\, c'\, () \\
\textbf{recv1}\, c'; & \\
\{\ell \mapsto 42\} & \\
\textbf{let}\, x = !\,\ell\, \textbf{in}\, \textbf{free}\, \ell; & \\
\{x = 42\} & \\
\textbf{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell})$$
$$\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\ c = \mathbf{new1}\,()\ \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathsf{chan\_prot}\} \\
\mathbf{let}\ \ell = \mathbf{ref}\,40\ \mathbf{in} \\
\{c \rightarrowtail \mathsf{chan\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\ c' = \mathbf{new1}\,()\ \mathbf{in}\ \mathbf{send1}\,c\,(\ell, c'); \\
\{c' \rightarrowtail (\mathsf{chan\_prot}'\ \ell)\} \\
\mathbf{recv1}\,c'; \\
\{\ell \mapsto 42\} \\
\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\,\ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathsf{chan\_prot}}\} \\
\mathbf{let}\ (\ell, c') = \mathbf{recv1}\,c\ \mathbf{in} \\
\{c' \rightarrowtail \overline{(\mathsf{chan\_prot}'\ \ell)} * \ell \mapsto 40\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \overline{(\mathsf{chan\_prot}'\ \ell)} * \ell \mapsto 42\} \\
\mathbf{send1}\,c'\,()
\end{array}
\right)
\end{array}
$$

One-shot channel protocols:

$$\mathsf{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathsf{chan\_prot}'\ \ell})$$
$$\mathsf{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$\{\mathsf{True}\}$
let $c =$ **new1** () **in**
$\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\}$

$\Big($
$\{c \rightarrowtail \mathtt{chan\_prot}\}$
let $\ell =$ **ref** 40 **in**
$\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\}$
let $c' =$ **new1** () **in** **send1** $c\,(\ell, c')$;
$\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell)\}$
recv1 c';
$\{\ell \mapsto 42\}$
let $x = !\,\ell$ **in** **free** ℓ;
$\{x = 42\}$
assert$(x = 42)$
$\{\mathsf{True}\}$

$\Big\|$

$\{c \rightarrowtail \overline{\mathtt{chan\_prot}}\}$
let $(\ell, c') =$ **recv1** c **in**
$\{c' \rightarrowtail \overline{(\mathtt{chan\_prot}'\ \ell)} * \ell \mapsto 40\}$
$\ell \leftarrow (!\,\ell + 2)$;
$\{c' \rightarrowtail \overline{(\mathtt{chan\_prot}'\ \ell)} * \ell \mapsto 42\}$
send1 c' ()
$\{\mathsf{True}\}$

$\Big)$

One-shot channel protocols:

$$\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell})$$
$$\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$\{\mathsf{True}\}$
$\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in}$
$\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\}$

$\{c \rightarrowtail \mathtt{chan\_prot}\}$	$\{c \rightarrowtail \overline{\mathtt{chan\_prot}}\}$
$\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in}$	$\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in}$
$\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\}$	$\{c' \rightarrowtail (\overline{\mathtt{chan\_prot}'\ \ell}) * \ell \mapsto 40\}$
$\mathbf{let}\, c' = \mathbf{new1}\,()\, \mathbf{in}\, \mathbf{send1}\, c\, (\ell, c');$	$\ell \leftarrow (!\,\ell + 2);$
$\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell)\}$	$\{c' \rightarrowtail (\overline{\mathtt{chan\_prot}'\ \ell}) * \ell \mapsto 42\}$
$\mathbf{recv1}\, c';$	$\mathbf{send1}\, c'\, ()$
$\{\ell \mapsto 42\}$	$\{\mathsf{True}\}$
$\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell;$	
$\{x = 42\}$	
$\mathbf{assert}(x = 42)$	
$\{\mathsf{True}\}$	

$\{\mathsf{True}\}$

One-shot channel protocols:

$$\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell})$$
$$\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

Questions?

Break (10 min!)

Dependent separation protocols

Tutorial Timeline

Part 1: 14:00 – 15:30

- ~~Introduction (10 min)~~
- ~~Layered implementation of session channels (10 min)~~
- ~~Basic concurrent separation logic and one-shot protocols (30 min)~~
- ~~**Break** (10 min)~~
- Dependent separation protocols (30 min)

Break (30 min)

Part 2: 16:00 – 17:30

- Iris invariants and ghost state (30 min)
- **Break** (10 min)
- Supervised Coq hacking (50 min)

Overview of Abstraction Layers

Layer	**Reasoning principles / specifications**
#1 Iris's HeapLang	Basic concurrent separation logic Iris invariants and ghost state
#2 One-shot channels	One-shot protocols
#3 Functional session channels	Dependent separation protocols
#4 Session channels	Dependent separation protocols

Functional Session Channels

Implementation (inspired by Kobayashi et al., Dardha et al.):

$$\mathbf{new}_{\mathbf{fun}}\,() \triangleq \mathbf{new1}\,()$$

$$\mathbf{send}\,c\,v \triangleq \mathbf{let}\,c' = \mathbf{new1}\,()\,\mathbf{in}\,\mathbf{send1}\,c\,(v, c');\ c' \qquad \mathbf{close}\,c \triangleq \mathbf{send1}\,c\,()$$

$$\mathbf{recv}\,c \triangleq \mathbf{recv1}\,c \qquad \mathbf{wait}\,c \triangleq \mathbf{recv1}\,c$$

Example program:

$$\texttt{ses\_fun\_ref\_prog} \triangleq \mathbf{let}\,c = \mathbf{new}_{\mathbf{fun}}\,()\,\mathbf{in}\begin{pmatrix}\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\ \mathbf{let}\,c' = \mathbf{send}\,c\,\ell\,\mathbf{in} \\ \mathbf{wait}\,c'; \mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\ \mathbf{assert}(x = 42)\end{pmatrix} \Bigg\| \begin{pmatrix}\mathbf{let}\,(\ell, c') = \mathbf{recv}\,c\,\mathbf{in} \\ \ell \leftarrow (!\,\ell + 2); \mathbf{close}\,c'\end{pmatrix}$$

Functional Session Channel Specifications?

Implementation (inspired by Kobayashi et al., Dardha et al.):

$$\mathbf{new}_{\mathtt{fun}}\,() \triangleq \mathbf{new1}\,()$$

$$\mathbf{send}\,c\,v \triangleq \mathbf{let}\,c' = \mathbf{new1}\,()\,\mathbf{in}\,\mathbf{send1}\,c\,(v, c');\ c' \qquad \mathbf{close}\,c \triangleq \mathbf{send1}\,c\,()$$

$$\mathbf{recv}\,c \triangleq \mathbf{recv1}\,c \qquad \mathbf{wait}\,c \triangleq \mathbf{recv1}\,c$$

Specifications:

$$\{\mathsf{True}\}\ \mathbf{new}_{\mathtt{fun}}\,()\ \{w.\ \exists c.\ w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$$

$$\{c \rightarrowtail ??? * ???\}\ \mathbf{send}\,c\,v\ \{w.\ ???\}$$

$$\{c \rightarrowtail ???\}\ \mathbf{recv}\,c\ \{w.\ ???\}$$

$$\{c \rightarrowtail ??? * ???\}\ \mathbf{close}\,c\ \{w.\ ???\}$$

$$\{c \rightarrowtail ???\}\ \mathbf{wait}\,c\ \{w.\ ???\}$$

Dependent Separation Protocols

$$\texttt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\texttt{chan\_prot}'\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\texttt{chan\_prot}'\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\texttt{chan\_prot}'\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot}\ (p : \mathsf{Loc} \to \mathsf{iProto}) \triangleq$$
$$(\mathsf{Send}, \lambda w.\ \exists(\ell : \mathsf{Loc}), c'.\ w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{p\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot}\ (P : \mathsf{Loc} \to \mathsf{iProp})\ (p : \mathsf{Loc} \to \mathsf{iProto}) \triangleq \\ (\mathsf{Send}, \lambda w.\ \exists(\ell : \mathsf{Loc}), c'.\ w = (\ell, c') * P\ \ell * c' \rightarrowtail \overline{p\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot}\ (v : \mathrm{Loc} \to \mathrm{Val})\ (P : \mathrm{Loc} \to \mathrm{iProp})\ (p : \mathrm{Loc} \to \mathrm{iProto}) \triangleq$$
$$(\mathsf{Send}, \lambda w.\ \exists(\ell : \mathrm{Loc}), c'.\ w = (v\ \ell, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$

Dependent Separation Protocols

$$\texttt{send\_prot}\ (\tau : \mathsf{Type})\ (v : \tau \to \mathsf{Val})\ (P : \tau \to \mathsf{iProp})\ (p : \tau \to \mathsf{iProto}) \triangleq$$
$$(\mathsf{Send}, \lambda w.\ \exists (x : \tau), c'.\ w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq$$
$$(\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq !\,(x : \tau)\,\langle v \rangle\{P\}.\,\overline{p}$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{\overline{p\ x}})$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$\texttt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$\texttt{close\_prot}\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$\texttt{close\_prot}\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Send}, \lambda w.\, w = () * \ell \mapsto 42)$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$\texttt{close\_prot}\ (P : \mathsf{iProp}) \triangleq (\mathsf{Send}, \lambda w.\, w = () * P)$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$!\,\mathbf{end}\{P\} \triangleq (\mathsf{Send}, \lambda w.\, w = () * P)$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$!\,\mathbf{end}\{P\} \triangleq (\mathsf{Send}, \lambda w.\, w = () * P)$$
$$?\mathbf{end}\{P\} \triangleq \overline{!\,\mathbf{end}\{P\}}$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$!\,\mathbf{end}\{P\} \triangleq (\mathsf{Send}, \lambda w.\, w = () * P)$$
$$?\mathbf{end}\{P\} \triangleq (\mathsf{Recv}, \lambda w.\, w = () * P)$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$?(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$!\,\mathbf{end}\{P\} \triangleq (\mathsf{Send}, \lambda w.\, w = () * P)$$
$$?\mathbf{end}\{P\} \triangleq (\mathsf{Recv}, \lambda w.\, w = () * P)$$

$$\texttt{chan\_prot} \triangleq !\,(\ell : \mathsf{Loc})\,\langle \ell\rangle\{\ell \mapsto 40\}.\,?\mathbf{end}\{\ell \mapsto 42\}$$

Functional Session Channels Specifications!

Dependent session protocols:

$$\begin{aligned}
\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p &\triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\,x, c') * (P\,x) * c' \rightarrowtail \overline{p\,x}) \\
\mathbf{?}(x : \tau)\,\langle v\rangle\{P\}.\,p &\triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\,x, c') * (P\,x) * c' \rightarrowtail p\,x) \\
\mathbf{!end}\{P\} &\triangleq (\mathsf{Send}, \lambda w.\, w = () * P) \\
\mathbf{?end}\{P\} &\triangleq (\mathsf{Recv}, \lambda w.\, w = () * P)
\end{aligned}$$

Functional session channel specifications:

$$\begin{aligned}
\{\mathsf{True}\}\ \ & \mathtt{new}_{\mathtt{fun}}\,()\ \ \{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\} \\
\{c \rightarrowtail (\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p) * P\,t\}\ \ & \mathtt{send}\,c\,(v\,t)\ \ \{w.\, \exists c'.\, w = c' * c' \rightarrowtail p\,t\} \\
\{c \rightarrowtail (\mathbf{?}\,(x : \tau)\,\langle v\rangle\{P\}.\,p)\}\ \ & \mathtt{recv}\,c\ \ \{w.\, \exists (x : \tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail p\,x\} \\
\{c \rightarrowtail \mathbf{!end}\{P\} * P\}\ \ & \mathtt{close}\,c\ \ \{\mathsf{True}\} \\
\{c \rightarrowtail \mathbf{?end}\{P\}\}\ \ & \mathtt{wait}\,c\ \ \{P\}
\end{aligned}$$

Proof of Send Specification

$$
\begin{array}{l}
\mathtt{let}\, c' = \mathtt{new1}\,() \,\mathtt{in} \\
\mathtt{send1}\, c\, (v\ t, c'); \\
c'
\end{array}
$$

Proof of Send Specification

$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \quad * \quad P\,t\}$
$\mathbf{let}\,c' = \mathbf{new1}\,()\,\mathbf{in}$
$\mathbf{send1}\,c\,(v\,t, c');$
c'

Proof of Send Specification

$$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \quad * \quad P\,t\}$$
$$\mathbf{let}\; c' = \mathbf{new1}\,()\;\mathbf{in}$$
$$\mathbf{send1}\,c\,(v\;t, c');$$
$$c'$$

Send protocol:

$$\mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists (x : \tau), c'.\,w = (v\;x, c') * (P\;x) * c' \rightarrowtail \overline{p\;x})$$

Proof of Send Specification

$$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \quad * \quad P\,t\}$$
$$\mathbf{let}\; c' = \mathbf{new1}\,()\;\mathbf{in}$$
$$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \quad * \quad P\,t \quad * \quad c' \rightarrowtail p\,t \quad * \quad c' \rightarrowtail \overline{p\,t}\}$$
$$\mathbf{send1}\,c\,(v\,t, c');$$
$$c'$$

Send protocol:

$$\mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists (x : \tau), c'.\,w = (v\,x, c') * (P\,x) * c' \rightarrowtail \overline{p\,x})$$

Proof of Send Specification

$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p \quad * \quad P\ t\}$
$\mathbf{let}\ c' = \mathbf{new1}\,()\ \mathbf{in}$
$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p \quad * \quad P\ t \quad * \quad c' \rightarrowtail p\ t \quad * \quad c' \rightarrowtail \overline{p\ t}\}$
$\mathbf{send1}\,c\,(v\ t, c');$
$\{c' \rightarrowtail p\ t\}$
c'

Send protocol:

$$\mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * (P\ x) * c' \rightarrowtail \overline{p\ x})$$

Proof of Send Specification

$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p \quad * \quad P\ t\}$
$\mathbf{let}\ c' = \mathbf{new1}\,()\ \mathbf{in}$
$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p \quad * \quad P\ t \quad * \quad c' \rightarrowtail p\ t \quad * \quad c' \rightarrowtail \overline{p\ t}\}$
$\mathbf{send1}\,c\,(v\ t, c');$
$\{c' \rightarrowtail p\ t\}$
c'
$\{w.\, \exists c'.\, w = c' * c' \rightarrowtail p\ t\}$

Send protocol:

$$\mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * (P\ x) * c' \rightarrowtail \overline{p\ x})$$

Functional Session Channel Example

$$
\begin{array}{l}
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, ()\, \mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell\, \mathbf{in} \\
\mathbf{wait}\, c'; \\
\mathbf{let}\, x = !\, \ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right)
\;\Big\|\;
\left(\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c\, \mathbf{in} \\
\ell \leftarrow (!\, \ell + 2); \\
\mathbf{close}\, c'
\end{array}\right)
\end{array}
$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\mathbf{wait}\, c'; \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
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\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
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\end{array}
$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
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\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
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\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{close}\, c'
\end{array}\right)
\end{array}
$$

Protocol:

$$\texttt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\mathbf{wait}\, c'; \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right)
\;\Big\|\;
\left(\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{close}\, c'
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\, c = \textbf{new}_{\textbf{fun}}\, () \,\textbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\textbf{let}\, \ell = \textbf{ref}\, 40 \,\textbf{in} \\
\textbf{let}\, c' = \textbf{send}\, c\, \ell \,\textbf{in} \\
\textbf{wait}\, c'; \\
\textbf{let}\, x = !\,\ell \,\textbf{in}\, \textbf{free}\, \ell; \\
\textbf{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\textbf{let}\, (\ell, c') = \textbf{recv}\, c \,\textbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\textbf{close}\, c'
\end{array}
\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\mathbf{wait}\, c'; \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right. \\
\left\| \begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{close}\, c'
\end{array}\right.
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathbf{wait}\, c'; \\
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\end{array}\right. \quad \Big\| \quad \left.\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
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\ell \leftarrow (!\,\ell + 2); \\
\mathbf{close}\, c'
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{ses\_prot}\} & \{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} & \mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} & \mathbf{close}\, c' \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} & \\
\mathbf{wait}\, c'; & \\
\{\ell \mapsto 42\} & \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; & \\
\mathbf{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathbf{wait}\, c'; \\
\{\ell \mapsto 42\} \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42)
\end{array}\right) \\
\quad \Big\| \\
\left(\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{close}\, c'
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\,() \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathbf{wait}\, c'; \\
\{\ell \mapsto 42\} \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{close}\, c'
\end{array}
\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\, c = \textbf{new}_{\textbf{fun}}\, () \,\textbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\textbf{let}\, \ell = \textbf{ref}\, 40 \,\textbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\textbf{let}\, c' = \textbf{send}\, c\, \ell \,\textbf{in} \\
\{c' \rightarrowtail \textbf{?end}\{\ell \mapsto 42\}\} \\
\textbf{wait}\, c'; \\
\{\ell \mapsto 42\} \\
\textbf{let}\, x = !\,\ell \,\textbf{in}\, \textbf{free}\, \ell; \\
\{x = 42\} \\
\textbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\textbf{let}\, (\ell, c') = \textbf{recv}\, c \,\textbf{in} \\
\{c' \rightarrowtail \textbf{!end}\{\ell \mapsto 42\} * \ell \mapsto 40\} \\
\ell \leftarrow (!\,\ell + 2); \\
\textbf{close}\, c'
\end{array}
\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \textbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \textbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{ses\_prot}\} & \{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} & \mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} & \{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto 42\} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} & \ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} & \{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto 42\} * \ell \mapsto 42\} \\
\mathbf{wait}\, c'; & \mathbf{close}\, c' \\
\{\ell \mapsto 42\} & \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; & \\
\{x = 42\} & \\
\mathbf{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathbf{wait}\, c'; \\
\{\ell \mapsto 42\} \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto 42\} * \ell \mapsto 40\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto 42\} * \ell \mapsto 42\} \\
\mathbf{close}\, c' \\
\{\mathsf{True}\}
\end{array}
\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathbf{wait}\, c'; \\
\{\ell \mapsto 42\} \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto 42\} * \ell \mapsto 40\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto 42\} * \ell \mapsto 42\} \\
\mathbf{close}\, c' \\
\{\mathsf{True}\}
\end{array}
\right) \\
\{\mathsf{True}\}
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, () \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40 \,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathbf{wait}\, c'; \\
\{\ell \mapsto 42\} \\
\mathbf{let}\, x = !\,\ell \,\mathbf{in}\, \mathbf{free}\, \ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c \,\mathbf{in} \\
\{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto x\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto (x+2)\} \\
\mathbf{close}\, c' \\
\{\mathsf{True}\}
\end{array}
\right) \\
\{\mathsf{True}\}
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\, \langle \ell \rangle \{\ell \mapsto x\}.\, \mathbf{?end}\{\ell \mapsto (x+2)\}$$

Session Channels Specifications

$$\mathbf{new}\,() \triangleq \mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\,() \,\mathbf{in}\, (\mathbf{ref}\, c, \mathbf{ref}\, c)$$

$$c.\mathbf{send}(v) \triangleq c \leftarrow \mathbf{send}\,(!\,c)\, v \qquad c.\mathbf{close}() \triangleq \mathbf{close}\,(!\,c); \mathbf{free}\, c$$

$$c.\mathbf{recv}() \triangleq \mathbf{let}\,(v, c') = \mathbf{recv}\,!\,c \,\mathbf{in}\, c \leftarrow c'; v \qquad c.\mathbf{wait}() \triangleq \mathbf{wait}\,(!\,c); \mathbf{free}\, c$$

$$c \overset{\mathsf{imp}}{\rightarrowtail} p \triangleq \exists (c' : \mathsf{Val}).\, c \mapsto c' * c' \rightarrowtail p$$

Session Channels Specifications

$$\mathbf{new}\,() \triangleq \mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\,()\,\mathbf{in}\,(\mathbf{ref}\, c, \mathbf{ref}\, c)$$

$$c.\mathbf{send}(v) \triangleq c \leftarrow \mathbf{send}\,(!\,c)\,v \qquad c.\mathbf{close}() \triangleq \mathbf{close}\,(!\,c); \mathbf{free}\, c$$

$$c.\mathbf{recv}() \triangleq \mathbf{let}\,(v, c') = \mathbf{recv}\,!\,c\,\mathbf{in}\, c \leftarrow c'; v \qquad c.\mathbf{wait}() \triangleq \mathbf{wait}\,(!\,c); \mathbf{free}\, c$$

$$c \rightarrowtail^{\mathsf{imp}} p \triangleq \exists (c' : \mathsf{Val}).\, c \mapsto c' * c' \rightarrowtail p$$

Actris specifications:

$$\{\mathsf{True}\}\ \mathbf{new}\,()\ \{w.\, \exists c_1, c_2.\, w = (c_1, c_2) * c_1 \rightarrowtail^{\mathsf{imp}} p * c_2 \rightarrowtail^{\mathsf{imp}} \overline{p}\}$$

$$\{c \rightarrowtail^{\mathsf{imp}} (\mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p) * P\, t\}\ c.\mathbf{send}(v\ t)\ \{c \rightarrowtail^{\mathsf{imp}} p\, t\}$$

$$\{c \rightarrowtail^{\mathsf{imp}} (\mathbf{?}(x : \tau)\,\langle v \rangle \{P\}.\, p)\}\ c.\mathbf{recv}()\ \{w.\, \exists (x : \tau).\, w = (v\, x) * P\, x * c \rightarrowtail^{\mathsf{imp}} p\, x\}$$

$$\{c \rightarrowtail \mathbf{!end}\{P\} * P\}\ c.\mathbf{close}()\ \{\mathsf{True}\}$$

$$\{c \rightarrowtail \mathbf{?end}\{P\}\}\ c.\mathbf{wait}()\ \{P\}$$

Session Channel Example

$$
\begin{array}{l}
\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\
c_1.\mathbf{send}(\ell); \\
c_1.\mathbf{wait}(); \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\
\mathbf{assert}(x = 42)
\end{array}\right)
\;\Big\|\;
\left(\begin{array}{l}
\mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\mathbf{close}()
\end{array}\right)
\end{array}
$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\
c_1.\mathbf{send}(\ell); \\
c_1.\mathbf{wait}(); \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\
\mathbf{assert}(x = 42)
\end{array}\right)
\;\Big\|\;
\left(\begin{array}{l}
\mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\mathbf{close}()
\end{array}\right)
\end{array}
$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\,(c_1, c_2) = \textbf{new}\,()\,\textbf{in} \\
\left(\begin{array}{l}
\textbf{let}\,\ell = \textbf{ref}\,40\,\textbf{in} \\
c_1.\textbf{send}(\ell); \\
c_1.\textbf{wait}(); \\
\textbf{let}\,x = !\,\ell\,\textbf{in}\,\textbf{free}\,\ell; \\
\textbf{assert}(x = 42)
\end{array}\right. \;\Big\|\; \left(\begin{array}{l}
\textbf{let}\,\ell = c_2.\textbf{recv}()\,\textbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\textbf{close}()
\end{array}\right)
\end{array}
$$

Protocol:

$$\texttt{ses\_prot} \triangleq \textbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\textbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\
c_1.\mathbf{send}(\ell); \\
c_1.\mathbf{wait}(); \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\
\mathbf{assert}(x = 42)
\end{array}\right. \;\Big\|\; \left.\begin{array}{l}
\mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\mathbf{close}()
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l|l}
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot}\} & \{c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} & \mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
c_1.\mathbf{send}(\ell); & \ell \leftarrow (!\,\ell + 2); \\
c_1.\mathbf{wait}(); & c_2.\mathbf{close}() \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; & \\
\mathbf{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\,(c_1, c_2) = \textbf{new}\,()\,\textbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot}\} \\
\textbf{let}\,\ell = \textbf{ref}\,40\,\textbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\} \\
c_1.\textbf{send}(\ell); \\
c_1.\textbf{wait}(); \\
\textbf{let}\,x = !\,\ell\,\textbf{in}\,\textbf{free}\,\ell; \\
\textbf{assert}(x = 42)
\end{array}\right. \\
\left.\begin{array}{l}
\{c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\textbf{let}\,\ell = c_2.\textbf{recv}()\,\textbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\textbf{close}()
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x+2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\,(c_1, c_2) = \textbf{new}\,()\,\textbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot}\} \\
\textbf{let}\,\ell = \textbf{ref}\,40\,\textbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\} \\
c_1.\textbf{send}(\ell); \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \textbf{?end}\{\ell \mapsto 42\}\} \\
c_1.\textbf{wait}(); \\
\textbf{let}\,x = !\,\ell\,\textbf{in}\,\textbf{free}\,\ell; \\
\textbf{assert}(x = 42)
\end{array}\right.
\left\|\begin{array}{l}
\{c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\textbf{let}\,\ell = c_2.\textbf{recv}()\,\textbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\textbf{close}()
\end{array}\right.
\left.\begin{array}{l}\\\\\\\\\\\\\\\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\textbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\,(c_1, c_2) = \textbf{new}\,()\,\textbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot}\} \\
\textbf{let}\,\ell = \textbf{ref}\,40\,\textbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\} \\
c_1.\textbf{send}(\ell); \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \textbf{?end}\{\ell \mapsto 42\}\} \\
c_1.\textbf{wait}(); \\
\{\ell \mapsto 42\} \\
\textbf{let}\,x = !\,\ell\,\textbf{in}\,\textbf{free}\,\ell; \\
\textbf{assert}(x = 42)
\end{array}\right. \quad \Big\| \quad \left(\begin{array}{l}
\{c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\textbf{let}\,\ell = c_2.\textbf{recv}()\,\textbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\textbf{close}()
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\textbf{?end}\{\ell \mapsto (x+2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l|l}
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot}\} & \{c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} & \mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
c_1.\mathbf{send}(\ell); & c_2.\mathbf{close}() \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathbf{?end}\{\ell \mapsto 42\}\} & \\
c_1.\mathbf{wait}(); & \\
\{\ell \mapsto 42\} & \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; & \\
\{x = 42\} & \\
\mathbf{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\,(c_1, c_2) = \textbf{new}\,() \,\textbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l|l}
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot}\} & \{c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\textbf{let}\,\ell = \textbf{ref}\,40 \,\textbf{in} & \textbf{let}\,\ell = c_2.\textbf{recv}() \,\textbf{in} \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
c_1.\textbf{send}(\ell); & c_2.\textbf{close}() \\
\{c_1 \rightarrowtail^{\mathsf{imp}} \textbf{?end}\{\ell \mapsto 42\}\} & \\
c_1.\textbf{wait}(); & \\
\{\ell \mapsto 42\} & \\
\textbf{let}\,x = !\,\ell \,\textbf{in}\,\textbf{free}\,\ell; & \\
\{x = 42\} & \\
\textbf{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \textbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\textbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

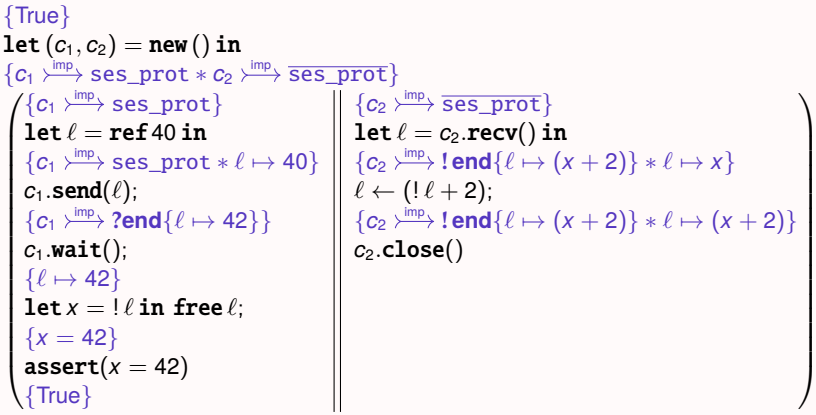
$\{\mathsf{True}\}$
$\mathbf{let}\ (c_1, c_2) = \mathbf{new}\,()\ \mathbf{in}$
$\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\}$

$\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot}\}$	$\{c_2 \rightarrowtail^{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\}$
$\mathbf{let}\ \ell = \mathbf{ref}\,40\ \mathbf{in}$	$\mathbf{let}\ \ell = c_2.\mathbf{recv}()\ \mathbf{in}$
$\{c_1 \rightarrowtail^{\mathsf{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\}$	$\{c_2 \rightarrowtail^{\mathsf{imp}} \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto x\}$
$c_1.\mathbf{send}(\ell);$	$\ell \leftarrow (!\,\ell + 2);$
$\{c_1 \rightarrowtail^{\mathsf{imp}} \mathbf{?end}\{\ell \mapsto 42\}\}$	$c_2.\mathbf{close}()$
$c_1.\mathbf{wait}();$	
$\{\ell \mapsto 42\}$	
$\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\,\ell;$	
$\{x = 42\}$	
$\mathbf{assert}(x = 42)$	
$\{\mathsf{True}\}$	

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x+2)\}$$

Session Channel Example



Protocol:

$$\texttt{ses\_prot} \triangleq \,!\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\ ?\mathbf{end}\{\ell \mapsto (x+2)\}$$

Session Channel Example

$\{\mathsf{True}\}$
$\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in}$
$\{c_1 \overset{\text{imp}}{\rightarrowtail} \mathtt{ses\_prot} * c_2 \overset{\text{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\}$

$\{c_1 \overset{\text{imp}}{\rightarrowtail} \mathtt{ses\_prot}\}$
$\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in}$
$\{c_1 \overset{\text{imp}}{\rightarrowtail} \mathtt{ses\_prot} * \ell \mapsto 40\}$
$c_1.\mathbf{send}(\ell);$
$\{c_1 \overset{\text{imp}}{\rightarrowtail} \mathbf{?end}\{\ell \mapsto 42\}\}$
$c_1.\mathbf{wait}();$
$\{\ell \mapsto 42\}$
$\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell;$
$\{x = 42\}$
$\mathbf{assert}(x = 42)$
$\{\mathsf{True}\}$

$\{c_2 \overset{\text{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\}$
$\mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in}$
$\{c_2 \overset{\text{imp}}{\rightarrowtail} \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto x\}$
$\ell \leftarrow (!\,\ell + 2);$
$\{c_2 \overset{\text{imp}}{\rightarrowtail} \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto (x+2)\}$
$c_2.\mathbf{close}()$
$\{\mathsf{True}\}$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x+2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\ (c_1, c_2) = \mathbf{new}\,()\ \mathbf{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot}\} \\
\mathbf{let}\ \ell = \mathbf{ref}\,40\ \mathbf{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * \ell \mapsto 40\} \\
c_1.\mathbf{send}(\ell); \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathbf{?end}\{\ell \mapsto 42\}\} \\
c_1.\mathbf{wait}(); \\
\{\ell \mapsto 42\} \\
\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\,\ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\ \ell = c_2.\mathbf{recv}()\ \mathbf{in} \\
\{c_2 \overset{\mathsf{imp}}{\rightarrowtail} \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto x\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c_2 \overset{\mathsf{imp}}{\rightarrowtail} \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto (x+2)\} \\
c_2.\mathbf{close}() \\
\{\mathsf{True}\}
\end{array}
\right) \\
\{\mathsf{True}\}
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}(\ell : \mathsf{Loc}, x : \mathbb{Z})\ \langle \ell \rangle \{\ell \mapsto x\}.\ \mathbf{?end}\{\ell \mapsto (x+2)\}$$

Break (30 min!)
We start again at 16:00!

If you attend the Coq hacking session please pull
`https://gitlab.mpi-sws.org/iris/tutorial-popl24`
and follow the installation instructions!

Iris invariants and ghost state

Tutorial Timeline

Part 1: 14:00 – 15:30

- ~~Introduction (10 min)~~
- ~~Layered implementation of session channels (10 min)~~
- ~~Basic concurrent separation logic and one-shot protocols (30 min)~~
- ~~**Break** (10 min)~~
- ~~Dependent separation protocols (30 min)~~

~~**Break** (30 min)~~

Part 2: 16:00 – 17:30

- Iris invariants and ghost state (30 min)
- **Break** (10 min)
- Supervised Coq hacking (50 min)

Overview of Abstraction Layers

Layer	**Reasoning principles / specifications**
#1 Iris's HeapLang	Basic concurrent separation logic Iris invariants and ghost state
#2 One-shot channels	One-shot protocols
#3 Functional session channels	Dependent separation protocols
#4 Session channels	Dependent separation protocols

One-Shot Channels Recap

One-shot channel implementations:

$$
\begin{aligned}
\texttt{new1}\,() &\triangleq \texttt{ref None}\\
\texttt{send1}\,c\,v &\triangleq c \leftarrow \texttt{Some}\,v\\
\texttt{recv1}\,c &\triangleq \texttt{let}\,x = !\,c\,\texttt{in}\\
&\quad\ \texttt{match}\,x\,\texttt{with}\\
&\quad\ \ \texttt{None} \Rightarrow \texttt{recv1}\,c\\
&\quad\ \mid \texttt{Some}\,v \Rightarrow \texttt{free}\,c;\ v\\
&\quad\ \texttt{end}
\end{aligned}
$$

One-shot channel specifications:

$$\{\mathsf{True}\}\ \texttt{new1}\,()\ \{w.\ \exists c.\ w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$$

$$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\,v\}\ \texttt{send1}\,c\,v\ \{\mathsf{True}\}$$

$$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}\ \texttt{recv1}\,c\ \{w.\ \Phi\,w\}$$

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Crux: Definition of $c \rightarrowtail p$

Definition of One-Shot Channel Resource

One-shot channel ownership defined using standard Iris methodology

$$c \rightarrowtail (tag, \Phi) \triangleq \ldots$$

Definition of One-Shot Channel Resource

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1. Model abstraction as a state transition system (STS)

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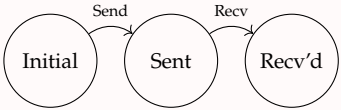

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One-shot channel ownership defined using standard Iris methodology:

1. Model abstraction as a state transition system (STS)
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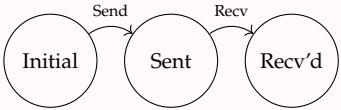

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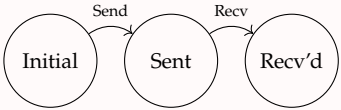

$\mathsf{chan\_inv} \triangleq (\underbrace{\quad}_{\text{(1) initial state}}) \lor (\underbrace{\quad}_{\text{(2) message sent, but not yet received}}) \lor (\underbrace{\quad}_{\text{(3) final state}})$

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One-shot channel ownership defined using standard Iris methodology:

1. Model abstraction as a state transition system (STS)
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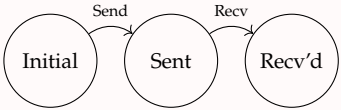
3. Determine resource ownership of each state



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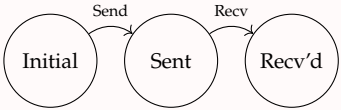

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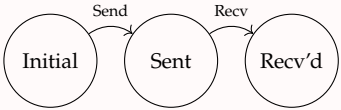

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One-shot channel ownership defined using standard Iris methodology:

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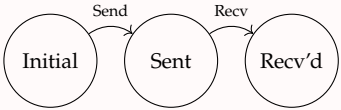

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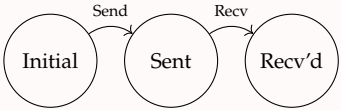

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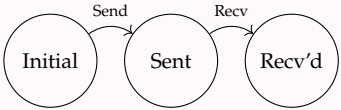

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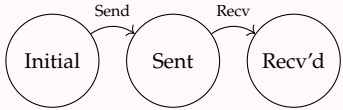

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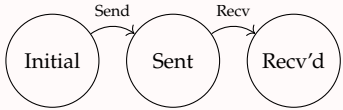

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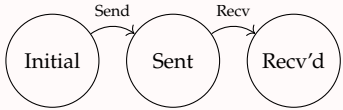

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Invariant duplication: $\boxed{R}^{\mathcal{N}} \vdash \boxed{R}^{\mathcal{N}} * \boxed{R}^{\mathcal{N}}$

Technicalities: names prevent opening the same invariant twice and the later $\triangleright$ is needed for impredicativity, i.e., $\boxed{\ldots \boxed{R}^{\mathcal{N}_2} \ldots}^{\mathcal{N}_1}$

Ghost Tokens

Consider the invariant:

$$\underbrace{(\, c \mapsto \texttt{None} \,)}_{\text{(1) initial state}} \vee \underbrace{(\, \exists v.\, c \mapsto \texttt{Some}\, v * \Phi\, v \ldots \,)}_{\text{(2) message sent, but not yet received}} \vee \underbrace{(\, \ldots \,)}_{\text{(3) final state}}$$

How to determine which state the one-shot channel is in?

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Ghost tokens allow deriving contradictions:

$$\mathsf{True} \Rrightarrow \exists \gamma.\, \mathsf{tok}\, \gamma \qquad \mathsf{tok}\, \gamma * \mathsf{tok}\, \gamma \vdash \mathsf{False}$$

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You can exclude cases which would end up in duplicate tokens:

$$\frac{((c \mapsto \mathbf{None}) \lor (\exists v.\, c \mapsto \mathbf{Some}\, v * \Phi\, v * \mathsf{tok}\, \gamma_s) \lor (\mathsf{tok}\, \gamma_s * \mathsf{tok}\, \gamma_r)) * \mathsf{tok}\, \gamma_s}{c \mapsto \mathbf{None} * \mathsf{tok}\, \gamma_s}$$

Proof of New

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

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$\{w.\, \exists c.\, w = c * c \mapsto \mathbf{None}\}$
$\{w.\, \exists c.\, w = c * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\{w.\, \exists c.\, w = c * \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$ // $p = (tag, \Phi)$
$\{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of New

$$\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq \underbrace{(\ c \mapsto \mathbf{None}\ )}_{\text{(1) initial state}} \lor \underbrace{(\exists v.\ c \mapsto \mathbf{Some}\ v * \Phi\ v * \text{tok}\ \gamma_s)}_{\text{(2) message sent, but not yet received}} \lor \underbrace{(\text{tok}\ \gamma_s * \text{tok}\ \gamma_r)}_{\text{(3) final state}}$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\ \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \text{tok}\ \gamma_s & \text{if } tag = \text{Send} \\ \text{tok}\ \gamma_r & \text{if } tag = \text{Recv} \end{cases}$$

$\{\text{True}\}$
ref None
$\{w.\ \exists c.\ w = c * c \mapsto \mathbf{None}\}$
$\{w.\ \exists c.\ w = c * c \mapsto \mathbf{None} * \text{tok}\ \gamma_s * \text{tok}\ \gamma_r\}$
$\{w.\ \exists c.\ w = c * \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_s * \text{tok}\ \gamma_r\}$ $\quad // p = (tag, \Phi)$
$\{w.\ \exists c.\ w = c * \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_s * \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_r\}$
$\{w.\ \exists c.\ w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of New

$$\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq \underbrace{(\ c \mapsto \mathbf{None}\ )}_{\text{(1) initial state}} \vee \underbrace{(\exists v.\ c \mapsto \mathbf{Some}\ v * \Phi\ v * \text{tok}\ \gamma_s)}_{\text{(2) message sent, but not yet received}} \vee \underbrace{(\text{tok}\ \gamma_s * \text{tok}\ \gamma_r)}_{\text{(3) final state}}$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\ \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \text{tok}\ \gamma_s & \text{if } tag = \text{Send} \\ \text{tok}\ \gamma_r & \text{if } tag = \text{Recv} \end{cases}$$

$\{\text{True}\}$
ref None
$\{w.\ \exists c.\ w = c * c \mapsto \mathbf{None}\}$
$\{w.\ \exists c.\ w = c * c \mapsto \mathbf{None} * \text{tok}\ \gamma_s * \text{tok}\ \gamma_r\}$
$\{w.\ \exists c.\ w = c * \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_s * \text{tok}\ \gamma_r\}$ // $p = (tag, \Phi)$
$\{w.\ \exists c.\ w = c * \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_s * \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_r\}$
$\{w.\ \exists c.\ w = c * c \rightarrowtail (\text{Send}, \Phi) * c \rightarrowtail (\text{Recv}, \Phi)\}$
$\{w.\ \exists c.\ w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$c \leftarrow \mathbf{Some}\ v$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}$
$c \leftarrow \mathbf{Some}\ v$
$\{\mathsf{True}\}$

Proof of Send

$$\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \text{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\text{tok}\ \gamma_s * \text{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \text{tok}\ \gamma_s & \text{if } tag = \text{Send} \\ \text{tok}\ \gamma_r & \text{if } tag = \text{Recv} \end{cases}$$

$\{c \rightarrowtail (\text{Send}, \Phi) * \Phi\ v\}$
$\{\boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_s * \Phi\ v\}$
$c \leftarrow \mathbf{Some}\ v$
$\{\text{True}\}$

Proof of Send

$$\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \textbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \textbf{Some}\ v * \Phi\ v * \text{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\text{tok}\ \gamma_s * \text{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \text{tok}\ \gamma_s & \text{if } tag = \text{Send} \\ \text{tok}\ \gamma_r & \text{if } tag = \text{Recv} \end{cases}$$

$\{c \rightarrowtail (\text{Send}, \Phi) * \Phi\ v\}$
$\{\boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_s * \Phi\ v\}$
  $\{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \text{tok}\ \gamma_s * \Phi\ v\}$
$c \leftarrow \textbf{Some}\ v$
$\{\text{True}\}$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathbf{None}}_{\text{(1) initial state}}) \lor (\underbrace{\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \lor (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (\mathit{tag}, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } \mathit{tag} = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } \mathit{tag} = \mathsf{Recv} \end{cases}$$

$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}$
$\{\boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$
  $\{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_s * \Phi\ v\}$
  $\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$
$c \leftarrow \mathbf{Some}\ v$
$\{\mathsf{True}\}$

Proof of Send

$$\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \textbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \textbf{Some}\ v * \Phi\ v * \text{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\text{tok}\ \gamma_s * \text{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \text{tok}\ \gamma_s & \text{if } tag = \text{Send} \\ \text{tok}\ \gamma_r & \text{if } tag = \text{Recv} \end{cases}$$

$\{c \rightarrowtail (\text{Send}, \Phi) * \Phi\ v\}$
$\{\boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_s * \Phi\ v\}$
  $\{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \text{tok}\ \gamma_s * \Phi\ v\}$
  $\{c \mapsto \textbf{None} * \text{tok}\ \gamma_s * \Phi\ v\}$
$c \leftarrow \textbf{Some}\ v$
  $\{c \mapsto \textbf{Some}\ v * \text{tok}\ \gamma_s * \Phi\ v\}$
$\{\text{True}\}$

Proof of Send

$$\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \textbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \textbf{Some}\ v * \Phi\ v * \text{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\text{tok}\ \gamma_s * \text{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \text{tok}\ \gamma_s & \text{if } tag = \text{Send} \\ \text{tok}\ \gamma_r & \text{if } tag = \text{Recv} \end{cases}$$

$\{c \rightarrowtail (\text{Send}, \Phi) * \Phi\ v\}$
$\{\boxed{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \text{tok}\ \gamma_s * \Phi\ v\}$
  $\{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \text{tok}\ \gamma_s * \Phi\ v\}$
  $\{c \mapsto \textbf{None} * \text{tok}\ \gamma_s * \Phi\ v\}$
$c \leftarrow \textbf{Some}\ v$
  $\{c \mapsto \textbf{Some}\ v * \text{tok}\ \gamma_s * \Phi\ v\}$
  $\{\text{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi\}$
$\{\text{True}\}$

Proof of Receive

```
let w = !c in
match w with
  None   ⇒ recv1 c
| Some v ⇒ free c; v
end
```

Proof of Receive

$\{c \rightarrowtail (\mathrm{Recv}, \Phi)\}$
let $w = !\,c$ **in**
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\, \Phi\, w\}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r * \boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}\}$
$\mathtt{let}\ w = !\,c\ \mathtt{in}$
$\mathtt{match}\ w\ \mathtt{with}$
$\quad \mathtt{None} \Rightarrow \mathtt{recv1}\ c$
$\mid \mathtt{Some}\ v \Rightarrow \mathtt{free}\ c;\ v$
$\mathtt{end}$
$\{w.\ \Phi\ w\}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
let $w = !\,c$ **in**
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\ \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
  $\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
let $w = !\,c$ **in**
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\ \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
  $\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
  $\{(c \mapsto \mathtt{None}) \vee (\exists v.\ c \mapsto \mathtt{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
$\mathtt{let}\ w = !\,c\ \mathtt{in}$
$\mathtt{match}\ w\ \mathtt{with}$
  $\mathtt{None} \Rightarrow \mathtt{recv1}\ c$
$|\ \mathtt{Some}\ v \Rightarrow \mathtt{free}\ c;\ v$
$\mathtt{end}$
$\{w.\ \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
  $\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
  $\{(c \mapsto \mathbf{None}) \vee (\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
  $\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$

```
let w = !c in
match w with
  None   ⇒ recv1 c
| Some v ⇒ free c; v
end
```

$\{w.\, \Phi\ w\}$

Duplicable propositions:

$\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
  $\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
  $\{(c \mapsto \mathbf{None}) \vee (\exists v.\ c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
  $\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
let $w = !\,c$ **in**
  $\{w = \mathbf{None} * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}$
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\ \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
  $\{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
  $\{(c \mapsto \mathbf{None}) \vee (\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
  $\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
let $w = !c$ **in**
  $\{w = \mathbf{None} * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}$
  $\{w = \mathbf{None} * \mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
match w **with**
 None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\, \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
  $\{(c \mapsto \textbf{None}) \vee (\exists v.\, c \mapsto \textbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
  $\{c \mapsto \textbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \textbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
let $w = !\,c$ **in**
  $\{w = \textbf{None} * c \mapsto \textbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
  $\{w = \textbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\, \Phi\ w\}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\text{Recv}, \Phi)\}$
$\{\texttt{tok}\ \gamma_r\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \texttt{tok}\ \gamma_r\}$
  $\{(c \mapsto \textbf{None}) \vee (\exists v.\, c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s) * \texttt{tok}\ \gamma_r\}$
  $\{c \mapsto \textbf{None} * \texttt{tok}\ \gamma_r\}\{c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s * \texttt{tok}\ \gamma_r\}$
let $w = !\,c$ **in**
  $\{w = \textbf{None} * c \mapsto \textbf{None} * \texttt{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s * \texttt{tok}\ \gamma_r\}$
  $\{w = \textbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \texttt{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \textbf{Some}\ v * \Phi\ v\}$
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\, \Phi\ w\}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\text{Recv}, \Phi)\}$
$\{\texttt{tok}\ \gamma_r\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \texttt{tok}\ \gamma_r\}$
  $\{(c \mapsto \textbf{None}) \lor (\exists v.\, c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s) * \texttt{tok}\ \gamma_r\}$
  $\{c \mapsto \textbf{None} * \texttt{tok}\ \gamma_r\}\{c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s * \texttt{tok}\ \gamma_r\}$
let $w = !\,c$ **in**
  $\{w = \textbf{None} * c \mapsto \textbf{None} * \texttt{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s * \texttt{tok}\ \gamma_r\}$
  $\{w = \textbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \texttt{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \textbf{Some}\ v * \Phi\ v\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * (w = \textbf{None} * \texttt{tok}\ \gamma_r) \lor (\exists v.\, w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v)\}$
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\, \Phi\ w\}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\text{Recv}, \Phi)\}$
$\{\texttt{tok}\ \gamma_r\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \texttt{tok}\ \gamma_r\}$
  $\{(c \mapsto \textbf{None}) \lor (\exists v.\ c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s) * \texttt{tok}\ \gamma_r\}$
  $\{c \mapsto \textbf{None} * \texttt{tok}\ \gamma_r\}\{c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s * \texttt{tok}\ \gamma_r\}$
let $w = !c$ **in**
  $\{w = \textbf{None} * c \mapsto \textbf{None} * \texttt{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v * \texttt{tok}\ \gamma_s * \texttt{tok}\ \gamma_r\}$
  $\{w = \textbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \texttt{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \textbf{Some}\ v * \Phi\ v\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * (w = \textbf{None} * \texttt{tok}\ \gamma_r) \lor (\exists v.\ w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v)\}$
$\{(w = \textbf{None} * \texttt{tok}\ \gamma_r) \lor (\exists v.\ w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v)\}$
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\ \Phi\ w\}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
  $\{(c \mapsto \mathbf{None}) \vee (\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
  $\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
let $w = !\,c$ **in**
  $\{w = \mathbf{None} * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
  $\{w = \mathbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \mathbf{Some}\ v * \Phi\ v\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * (w = \mathbf{None} * \mathsf{tok}\ \gamma_r) \vee (\exists v.\, w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v)\}$
$\{(w = \mathbf{None} * \mathsf{tok}\ \gamma_r) \vee (\exists v.\, w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v)\}$
$\{w = \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v\}$
match w **with**
  None $\Rightarrow$ **recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\, \Phi\ w\}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
  $\{(c \mapsto \mathbf{None}) \lor (\exists v.\ c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
  $\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
let $w = !\,c$ **in**
  $\{w = \mathbf{None} * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
  $\{w = \mathbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \mathbf{Some}\ v * \Phi\ v\}$
  $\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * (w = \mathbf{None} * \mathsf{tok}\ \gamma_r) \lor (\exists v.\ w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v)\}$
$\{(w = \mathbf{None} * \mathsf{tok}\ \gamma_r) \lor (\exists v.\ w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v)\}$
$\{w = \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v\}$
match w **with**
  None $\Rightarrow$ $\{\mathsf{tok}\ \gamma_r\}$**recv1** c
| **Some** $v \Rightarrow$ **free** c; v
end
$\{w.\ \Phi\ w\}$

Proof of Receive

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$\{w = \textbf{None} * \texttt{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v\}$
match w **with**
  None $\Rightarrow$ $\{\texttt{tok}\ \gamma_r\}\{c \rightarrowtail (\text{Recv}, \Phi)\}$**recv1** $c\{w.\ \Phi\ w\}$
| **Some** v $\Rightarrow$ $\{c \mapsto \textbf{Some}\ v * \Phi\ v\}$ **free** c; $v\{w.\ \Phi\ w\}$
end
$\{w.\ \Phi\ w\}$

Questions?

Iris and Actris Beyond This Tutorial

Iris

- **Features:** Custom ghost state, persistent modality, Löb induction, ...
- **Technicalities:** Later modality, invariant masks, ghost updates, ...
- Website: `https://iris-project.org`

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Actris

- Recursive protocols (POPL'20)
- Semantic Session Type System (CPP'21)
- Subprotocols (cf. subtyping) (LMCS'22)
- Dependent separation protocol ghost state and rules (LMCS'22)
- Application to distributed systems (ICFP'23)
- Deadlock-freedom (POPL'24 on Thursday: 14:40)
- Website: `https://iris-project.org/actris`

Break (10 min!)

Time for Coq hacking session!