Machine-Checked Semantic Session Typing

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Session Types – A Type System for Message Passing

Syntax

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\begin{array}{rll}
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   - Session type systems enforce a strict ownership discipline of channels
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$$\lambda c.\,(\texttt{recv}\ c \mid\mid \texttt{recv}\ c) : \texttt{chan}\ (\texttt{?Z}.\,\texttt{?Z}.\,\texttt{end}) \multimap (\mathrm{Z} \times \mathrm{Z})$$

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3. **Lack of mechanised soundness proofs for session type systems**
   - Few results exist for simpler systems
   - None exist for more expressive systems

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- Modernly defined in terms of a program logic
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Actris [Hinrichsen et al. POPL'20]

- **Dependent separation protocols:** Logical protocols inspired by session types
- Mechanised in **Coq**, with tactic support

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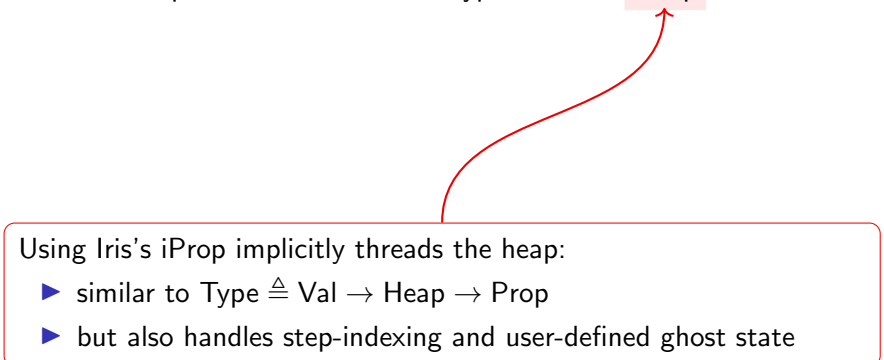

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Iris's *separation conjunction* $(P * Q)$ states that P and Q hold for disjoint parts of the heap

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Semantic Session Types

Session types as a new type kind:

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	Dependent separation protocols	**Session types**
Example	$\mathbf{?}\,(x : \mathbb{Z})\,\langle x \rangle \{\mathsf{True}\}.\,\mathbf{!}\,(y : \mathbb{Z})\,\langle y \rangle \{y = x + 2\}.\,\mathbf{end}$	$\mathbf{?}\mathsf{Z}.\,\mathbf{!}\mathsf{Z}.\,\mathtt{end}$
Usage	$c \rightarrowtail \mathit{prot}$	c : $\mathtt{chan}\ S$

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Session types as dependent separation protocols (iProto):

$$\begin{aligned} \mathrm{Type}_{\blacklozenge} &\triangleq \mathrm{iProto} \\ !A.\, S &\triangleq \mathbf{!}\,(v : \mathrm{Val})\,\langle v\rangle\{A\,v\}.\, S \\ ?A.\, S &\triangleq \mathbf{?}\,(v : \mathrm{Val})\,\langle v\rangle\{A\,v\}.\, S \\ \mathtt{end} &\triangleq \mathbf{end} \end{aligned} \qquad \begin{aligned} \mathrm{Type}_{\bigstar} &\triangleq \mathrm{Val} \to \mathrm{iProp} \\ \mathtt{chan}\ S &\triangleq \lambda\, w.\, w \rightarrowtail S \end{aligned}$$

Dependent separation protocols:

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Semantic Session Types – Example Proof

Rule:

$$\Gamma, (x : \texttt{chan}\ (?A.\, S)) \vDash \texttt{recv}\ x : A \dashv \Gamma, (x : \texttt{chan}\ S)$$

Proof:

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Lemma ltyped_recv Γ x A S :
  Γ !! x = Some (chan (<??> TY A; S))%lty →
  Γ ⊨ recv x : A ⊣ ctx_cons x (chan S) Γ.
Proof.
  iIntros (HΓx%ctx_lookup_perm) "!>".
  iIntros (σ) "HΓ /=". rewrite {1}HΓx /=.
  iDestruct (ctx_ltyped_cons with "HΓ") as
    (c Hσ) "[Hc HΓ]".
  rewrite Hσ.
  wp_recv (v) as "HA".
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Qed.
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- Message-passing is handled using **Actris**
- Mechanised in Coq using the **Iris Proof Mode**

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3. **Mechanised soundness proof of our results**
   - We mechanised it in Coq: https://gitlab.mpi-sws.org/iris/actris/-/tree/cpp21
   - By building on top of Iris and Actris frameworks and libraries
   - Stable artifact: https://zenodo.org/record/4322752